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The Simple Inhomogeneous Model: Chemical Evolution of Metal-poor Systems

M. S. OEY
Lowell Observatory

Abstract

A Simple Inhomogeneous Model (SIM) for galactic chemical evolution is offered as a standard reference model in the stochastic, low-metallicity regime where the homogeneous Simple Model breaks down. In this model, a parent metallicity distribution function (MDF) of contaminating events drives the chemical evolution of a system. This SIM model is applied to the MDF of the Galactic halo, yielding an interpretation of its evolution that differs from the application of the Simple Model. The SIM also predicts the fraction of zero-metallicity Population III stars; the observed discrepancy can be likened to the G-dwarf Problem for the Galactic disk.

1.1 Inhomogeneous Evolution and Element Dispersal

Inhomogeneous models for galactic chemical evolution acknowledge the interstellar element dispersal processes. On a detailed, stochastic level, these dispersal processes determine the metallicity increments that drive the chemical evolution of the system. Dispersal can be considered in three regimes: 1. Local **mixing** of stellar products within a given region, corresponding to the influence of individual star-forming regions; 2. Global **homogenization** of the system, which works toward eliminating the differentials between the above regions; 3. **Outflow** from the system altogether (see review by Oey 2002). How these modes of element dispersal affect a system's chemical evolution depends on their relevant timescales with respect to the star-formation timescale.

Inhomogeneous chemical evolution models provide the greatest insight in the limit that opposes the homogeneous Simple Model, namely, in the regime of localized dispersal from individual star-forming regions. Here, we consider the individual units of metallicity δZ that stochastically drive chemical evolution of a system. In the earliest stages, the system's metallicity dispersion is large, of order δZ; whereas later, the dispersion decreases, as the metallicity $Z \gg \delta Z$ (e.g., Edmunds 1975). Thus the instantaneous metallicity dispersion σ_Z is an important diagnostic of chemical evolution. We also see that a threshold minimum metallicity $Z_{\rm min}$ is expected, which essentially corresponds to δZ (Audouze & Silk 1995).

Empirically, this decrease in σ_Z vs. Z is seen in long-lived stars of the Milky Way (e.g., McWilliam et al. 1995; Ryan et al. 1996). A low-metallicity threshold in the most metal-poor stars has also been suggested to exist around [Fe/H] ~ -4 (Beers 1999; but see Christlieb et al. 2002). More inhomogeneous models are therefore incorporating stochastic

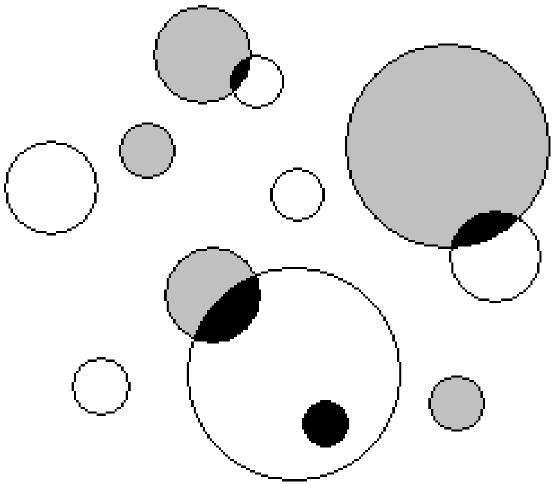

Fig. 1.1. Schematic cartoon of inhomogeneous evolution described by equations 1.1 to 1.3. Each set of j overlapping regions of contamination has its own MDF $N_j(Z)$. Here, the black regions correspond to $j = 2$. The white and gray regions represent different generations k, but both correspond to $j = 1$.

processes in order to better understand the metal-poor regime. For example, the patterns of metallicity scatter can be used to constrain numerical inhomogeneous models (Argast et al. 2000). Fields et al. (2002) also present a beautiful analysis of the σ_Z evolution and present examples of models that predict dramatically different metallicity patterns for the most metal-poor stars, depending on, for example, the relative magnitude of r-process vs. Fe production.

1.2 The Simple Inhomogeneous Model

We therefore see that inhomogeneous chemical evolution is required to understand the earliest enrichment events and most metal-poor systems. Inhomogeneous models in turn depend on understanding the localized dispersal and mixing processes that determine the metallicity units δZ. It is therefore desirable to have a simple, analytic model that describes global chemical evolution, and that both incorporates the stochastic enrichment process and parameterizes the element dispersal.

The Simple Inhomogeneous Model (SIM; Oey 2000, 2003) accommodates these characteristics and offers an opposing limit to the homogeneous Simple Model. The premise is that a constant parent metallicity distribution function (MDF) stochastically builds the total metallicity as the system evolves. From this parent MDF, $f(Z)$, we can derive the distributions $N_j(Z)$ that represent the MDFs for the sets of regions having j overlapping units of contamination (Figure 1.1). Thus, the instantaneous MDF after n generations of star formation is given by:

$$N_{\rm ISM}(Z) = \sum_{j=1}^{n} P_j\, N_j(Z) \quad , \tag{1.1}$$

where P_j is the probability of having j overlapping units of contamination, which is given by the binomial distribution:

$$P_j = \binom{n}{j} Q^j \, (1-Q)^{n-j} \quad , \quad 1 \leq j \leq n \quad . \tag{1.2}$$

Q is the constant filling factor occupied by each generation of contamination. Equation 1.1 thus represents the MDF of the ISM at any given time, or the MDF for young stars.

Similarly, the total MDF of all stars ever formed can be derived by:

$$N_{\rm tot}(Z) = \frac{1}{n} \sum_{j=1}^{n} \sum_{k=j}^{n} D_{k-1} \, P_{j,k} \, N_j(Z) \, , \tag{1.3}$$

where $P_{j,k}$ is the probability for generation k, of having j overlapping units, and D_{k-1} is the fraction of gas remaining after k generations. Equation 1.3 thus represents the MDF of long-lived stars. As described by Oey (2000), the product nQ describes the evolutionary state of the system; for large nQ, equation 1.3 approaches the MDF of the Simple Model.

It is apparent that $N_{\rm ISM}(Z)$ and $N_{\rm tot}(Z)$ are driven by the parent MDF, $f(Z)$, via the distributions $N_j(Z)$. The parent MDF thus parameterizes the metallicity units that drive the global chemical evolution of the system. While the SIM can accommodate any analytic form for $f(Z)$, we assumed:

$$f(Z) \propto Z^{-2} \quad , \quad Z_{\rm min} \leq Z \leq Z_{\rm max} \quad , \tag{1.4}$$

for Z determined by core-collapse supernova (SN) products. As described in more detail by Oey (2000, 2003), equation 1.4 is derived from the empirical power-law form of the H II region luminosity function seen in local star-forming galaxies. The power-law exponent of –2 results from arguments that dilute stellar products into the corresponding super-bubble volumes generated by the SNe.

For a parent $f(Z)$ of the form in equation 1.4, the SIM yields chemical evolution as shown in Figure 1.2. The instantaneous and total MDFs are shown after $n = 1$, 3, and 10 generations, for $Q = 0.8$. Minimum and maximum values of $-4.7 \leq Z \leq 3.0$ are assumed for $f(Z)$. As discussed below, the range in values for $f(Z)$ is difficult to estimate from first principles. The left column in the figure shows the evolution of the instantaneous MDF, $N_{\rm ISM}(Z)$, while the right column shows the total, time-integrated MDF of all stars ever created. The histograms show Monte Carlo models corresponding to stochastic drawings from $f(Z)$, while the curves represent analytic approximations based on the Central Limit theorem (see Oey 2000). The hatched bar in the left column shows the fraction of zero-metallicity gas that remains.

We see that the MDFs for the first generation of star formation naturally correspond to the parent $f(Z)$. As more generations take place, the simple power law distribution transforms. For the instantaneous MDF, it approaches a normal distribution; and for the total MDF, it approaches the Simple Model exponential, retaining a high-metallicity tail that shrinks as the system continues to evolve. Figure 1.2 emphasizes that, at the earliest stages through the transition regime, the forms of the MDF are sensitive to the form of $f(Z)$. As dictated by the Central Limit Theorem, the MDFs are relatively insensitive to $f(Z)$ beyond this stage.

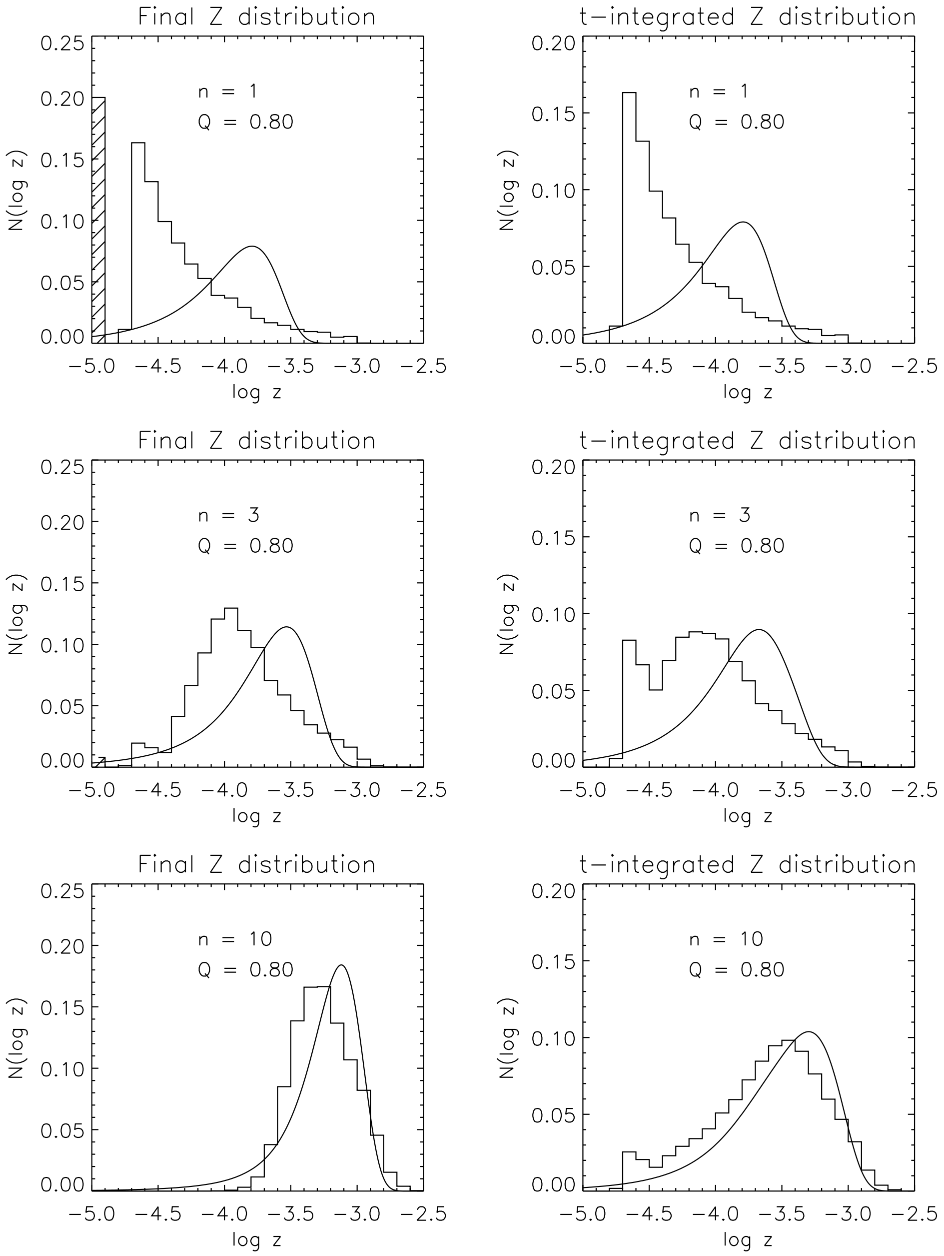

Fig. 1.2. Evolution of the MDFs predicted by the Simple Inhomogeneous Model. The left and right columns respectively show the instantaneous (equation 1.1) and time-integrated (equation 1.3) MDFs for $n = 1$, 3, and 10 generations at constant Q. The histograms show Monte Carlo models, and the curves show analytic approximations. The hatched bar in the left column shows the fraction of pristine gas remaining.

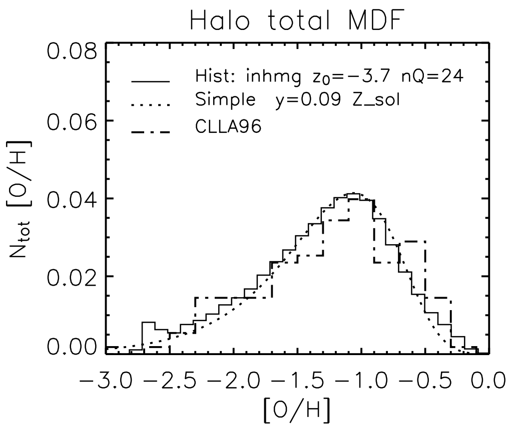

Fig. 1.3. The observed Galactic halo MDF from Carney et al. (1996; dot-dashed line) is shown overplotted by two models: the homogeneous Simple Model (dotted line); and a Simple Inhomogeneous Model having a mean value for the parent $f(Z)$ of $Z_0 = -3.7$ ([O/H] = –2.0; solid histogram)

1.3 The Galactic Halo

Figure 1.3 shows the SIM (solid histogram) applied to the Galactic halo MDF observed by Carney et al. (1996; dot-dashed line). The dotted line shows the homogeneous Simple Model. Oey (2003) describes the quantitative details of the models. It is apparent that both the homogeneous and inhomogeneous models agree well with the data. However, they correspond to dramatically different interpretations: the Simple Model shows a turnover at high Z because the system runs out of gas and cannot make many metal-rich stars; whereas for the SIM, the high-metallicity tail results from the decreasing power-law form of the parent $f(Z)$. Thus, for the homogeneous model, the halo MDF represents a fairly evolved system with respect to gas consumption; whereas for the SIM, it is still relatively unevolved in that regard. Oey (2003) presents a detailed discussion of various SIM models and the empirical halo MDF; here, the point is simply to emphasize that the SIM yields fundamental insights into the evolution and interpretation of metal-poor systems, that may differ significantly from the results of the homogeneous Simple Model.

1.4 The Fraction of Population III Stars

The very first star-formation events correspond to Population III stars, and are necessarily strongly dominated by stochastic events. Hence, the SIM offers powerful constraints in this regime. The fraction of zero-metallicity Population III stars predicted by the SIM is (Oey 2003):

$$F_{\rm III} = \sum_{k=1}^{n} D_{k-1} P_{1,k} \Bigg/ \sum_{j=1}^{n} \sum_{k=j}^{n} D_{k-1} P_{j,k} \quad . \tag{1.5}$$

The numerator corresponds to all stars created in pristine gas, including those created in generations subsequent to the first (e.g., both white and gray regions in Figure 1.1); the denominator corresponds to all stars ever created. Since, as mentioned above, the evolutionary state of the system is described by the product nQ, the value of $F_{\rm III}$ also depends on nQ, but is quite robust to the individual values of n and Q. Thus, for any SIM model, it is straightforward to estimate $F_{\rm III}$.

As discussed in detail by Oey (2003), the SIM models that best fit the Galactic halo MDF yield $F_{\rm III} \gtrsim 3 \times 10^{-2}$. This strongly contrasts with the present empirical limit of $< 4 \times 10^{-4}$. This discrepancy with the SIM can be likened to the G-dwarf discrepancy with the Simple Model: it clearly indicates that a fundamental process is not accounted for by this rudimentary analysis. It is also an interesting coincidence that the parallel with the G-dwarf Problem is also in the sense that the data fail to show enough zero-metallicity stars.

1.5 The Low-metallicity Threshold

As described in the first section, $Z_{\rm min}$ of the parent $f(Z)$ sets the minimum metallicity that should be observed in the system. Since the range of $f(Z)$ is determined by the interstellar mixing mechanisms, and also the SN yields, the value of $Z_{\rm min}$ can only be estimated if these dispersal processes are well-understood. Oey (2003) presents a crude analysis and de Avillez & Mac Low (2002) carry out some preliminary numerical simulations of interstellar mixing, but a reliable quantitative model is still greatly lacking.

Within the coarse constraints of these dispersal analyses, the present empirical limit of [Fe/H] ~ -4 appears to be consistent with $Z_{\rm min}$ for the range of $f(Z)$ required by the SIM halo models. We do note the exciting discovery by Christlieb et al. (2002) of a star with [Fe/H] ~ -5.3: it will be critical to determine whether the MDF decreases continuously to these metallicities, or whether this star is essentially an anomaly, with a strong drop-off in the MDF remaining near [Fe/H] ~ -4. If the latter remains true, then this value may indeed represent a threshold metallicity corresponding to $Z_{\rm min}$ of the parent $f(Z)$. In this case, anomalous values below $Z_{\rm min}$ occasionally may still exist, since element dispersal for individual regions is undoubtedly variable. However, if the MDF turns out to be continuous down to [Fe/H] ~ -5 and beyond, then clearly the estimated range, and probably also the form, of $f(Z)$ will need to be revised, to be consistent with both the halo MDF and observed low-metallicity threshold.

1.6 Conclusions

In summary, the Simple Inhomogeneous Model offers powerful constraints on both the chemical evolution and interstellar dispersal processes for the most unevolved, metal-poor systems. It emphasizes how the metallicity distribution $f(Z)$ of the parent enrichment events drives the chemical evolution of the system. In turn, $f(Z)$ is determined by the interstellar element dispersal processes, in concert with the stellar product yields.

Application of the SIM to the Galactic halo yields a fundamentally different interpretation of the halo evolution, implying a relatively unevolved system with respect to gas consumption. This contrasts with the implication from the homogeneous Simple Model that the halo is evolved in terms of gas consumption. The SIM also yields a straightforward prediction for the fraction of zero-metallicity Population III stars. There is a strong discrepancy between the predicted and observed fraction, similar in spirit to the well-known G-dwarf Problem for the Simple Model and the Galactic disk. Finally, we emphasize the importance of clarifying the present empirical low-metallicity threshold: the form of the MDF in this regime strongly constrains the range and form of $f(Z)$, which are both crucial parameters for inhomogeneous chemical evolution.

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