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Thermal Conductivity of the ICM

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Abstract

We demonstrate that electron thermal conduction in a weakly collisional plasma with turbulent chaotic magnetic fields approaches the Spitzer limit. Such high thermal conduction in the intra-cluster medium should certainly be important in dense cool cores of clusters. It provides additional heating needed to explain the absence multi-temperature gas in cooling flows.

1.1 Introduction

Strong, energetically important X-ray emission from many galaxy clusters was believed to indicate that a significant amount of mass must continuously cool and drop out of the intra-cluster medium (Fabian 1994). The mass deposition rate is estimated to be as much as several hundred $M_\odot\,\mathrm{yr}^{-1}$ in some clusters. However, recent observations with *XMM-Newton* and *Chandra* have failed to find the multi-temperature gas one expects in a cooling flow. The observations suggest that mass dropout may be less significant than previously thought. A reduced level of mass dropout is possible if there is a source of heat to replace the energy that is lost through X-ray emission, but no clear heat source has yet been identified.

The inner region of a cluster ($R \sim$ few $\times 10$ kpc) where mass dropout appears to be occurring is typically cooler than the rest of the cluster. Therefore, it seems natural that a source of heat may likely be thermal conduction from the hot outer regions of the cluster to the center. The argument against such heat source is based upon the general belief that magnetic fields (always present in the ICM) should drastically suppress thermal transport to an uninterestingly low level. However, if thermal conduction in a cluster is as efficient as in Spitzer's theory, or even if it is a factor of a few less efficient, heat conduction will have a strong effect on the energetics of a cooling flow and perhaps will shut off mass dropout.

1.2 Thermal Conduction Regimes

1.2.1 Unmagnetized Medium

In the absence of magnetic field the heat transport is diffusive (as sketched in Fig. 1.1a). Spitzer (1962) showed that thermal conduction in an unmagnetized plasma has a

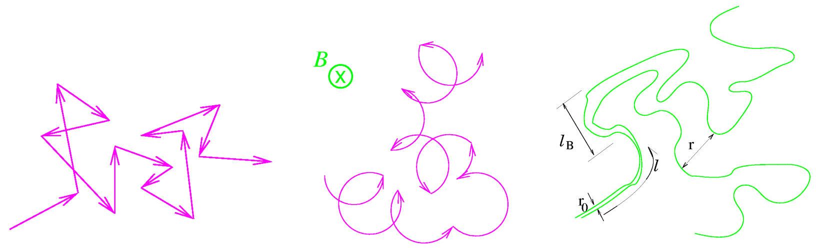

Fig. 1.1. The first panel (a) sketches the diffusive nature of heat transport in an unmagnetized plasma. Collisional transport in a homogeneous magnetic field is illustrated in the second panel (b) where the transverse displacement per collision is limited by the Larmor radius (usually $\rho_e \ll \lambda_{\rm mfp}$). Particle transport along chaotic field lines is shown in the last panel (c).

diffusion constant* $\kappa_{\rm Sp} \sim \lambda^2/t_{\rm Coul} = \lambda v_t \sim 4\times10^{32}\, T_1^{5/2} n_{-3}^{-1}$ cm^2 s^{-1}, where $t_{\rm Coul} = \lambda/v_t$ is the mean free time between Coulomb collisions, and λ and v_t are the mean free path and the thermal speed of electrons (Cowie & McKee 1977; Ettori & Fabian 2000): $\lambda \sim 30\, T_1^2 n_{-3}^{-1}$ kpc, $v_t \sim (kT/m_e)^{1/2} \sim 4\times10^9\, T_1^{1/2}$ cm s^{-1}. Here, $T_1 = kT/10$ keV is the scaled temperature, $n_{-3} = n/10^{-3}$ cm^{-3} is the scaled electron number density, and we have used an average value for the Coulomb logarithm, $\ln\Lambda \sim 38$. The time required for heat to diffuse conductively across a radius $R = 100R_2$ kpc is given by $t_{\rm Sp} \sim R^2/\kappa_{\rm Sp} \sim 8\times10^6\, T_1^{-5/2} n_{-3} R_2^2$ yr. For conduction to have a significant effect on a cooling flow, the conduction time must be comparable to the cooling time $t_{\rm cool}$ of the gas.

1.2.2 Homogeneous Magnetic Field

In the presence of an ordered magnetic field, conduction is anisotropic. Electrons stream freely parallel to the field line, so the parallel diffusion constant is almost equal to the Spitzer value: $\kappa_\parallel \sim \kappa_{\rm Sp}/3$. (The factor of $1/3$ is because diffusion is in one dimension rather than three). Perpendicular to the field, however, electrons follow circular Larmor orbits with radius $\rho_e \ll \lambda$. Since an electron moves only a distance $\sim \rho_e$ in each scattering (as in Fig. 1.1a), the perpendicular diffusion constant is given by $\kappa_\perp \sim \rho_e^2/t_{\rm Coul} \sim (\rho_e/\lambda)^2\kappa_{\rm Sp} \ll \kappa_{\rm Sp}$. For a galaxy cluster with a magnetic field of $\sim 10^{-6}$ G, we have $\rho_e \sim 10^{-12}\lambda$, and $\kappa_\perp$ is effectively zero.

1.2.3 Chaotic Magnetic Field

Thermal conduction behaves very differently when the magnetic field is chaotic. The theory for a tangled field with a single coherence length l_B was developed by Rechester & Rosenbluth (1978, [RR]) and has been recently revived in the astrophysical context by Chandran & Cowley (1998, [CC]). Since the field is chaotic, the separation r of two nearby field lines must have a Lyapunov-like scaling as a function of distance l along the field:

$$r \sim r_0 \exp(l/L_{\rm Lyap}), \tag{1.1}$$

* The coefficient κ_c which enters in the heat flux equation, $q = -\kappa_c \nabla T$, is related to $\kappa_{\rm Sp}$ by $\kappa_c = nk_B\kappa_{\rm Sp}$, where k_B is the Boltzmann constant. Note also that heat diffusion and particle diffusion have slightly different coefficients, differing by a factor of order unity.

where r_0 is the initial separation of the two lines, see Fig. 1.1c.. As there is only one characteristic scale in the problem, namely l_B, we expect $L_{\rm Lyap} \sim l_B$.

Let us consider the evolution of a compact cloud of electrons of initial size ρ_e. With time, the electrons diffuse parallel to the field, with a diffusion constant $\kappa_\|$. As the electron cloud spreads out, its perpendicular extent diverges exponentially according to equation (1.1). Thus, by the time the electrons have diffused a Rechester-Rosenbluth distance $L_{RR} \sim l_B \ln(l_B/\rho_e) \sim 30 l_B$ along the field line, their transverse separation is of order l_B. The numerical coefficient 30 corresponds to $l_B/\rho_e \sim 10^{13}$, a typical value for a galaxy cluster (assuming l_B is a fraction of the radius). Being a logarithmic factor, the numerical value is insensitive to details. When electrons have moved a distance L_{RR} along the tangled field line, their three-dimensional root mean square (rms) displacement is R_*, where $R_*^2 \sim L_{RR} l_B \sim 30 l_B^2$. Beyond R_*, the motion of an electron is isotropic and uncorrelated with its previous path. We define t_* as the time it takes for electrons to diffuse a distance L_{RR} along the field: $t_* \sim L_{RR}^2/\kappa_\|$. For $t < t_*$, electrons diffuse anisotropically: $l \sim (\kappa_\| t)^{1/2}, r \sim \rho_e \exp[(\kappa_\| t)^{1/2}/l_B]$. For $t > t_*$, however, electrons diffuse isotropically, and move in three dimensions according to $R \sim (\kappa_* t)^{1/2}$, where

$$\kappa_* \sim R_*^2/t_* \sim (l_B/L_{RR})\kappa_\| \sim 10^{-2}\kappa_{\rm Sp}. \tag{1.2}$$

We see that for $R > R_*$ the conduction is many orders of magnitude more efficient than when the field is ordered. However, κ_* is still a factor ~ 100 less than $\kappa_{\rm Sp}$. The conduction time is correspondingly ~ 100 times longer than the Spitzer time $t_{\rm Sp}$. The estimate given in equation (1.2) is valid so long as $\lambda < l_B$. This condition, which is likely to be satisfied by the gas in clusters, ensures that collisions enable electrons to pass through magnetic mirrors caused by inhomogeneities in the field. If $\lambda > l_B$, conduction is suppressed by an additional factor $\theta < 1$ since only a fraction of the electrons are able to penetrate the mirrors.

1.2.4 Turbulent Magnetic Field

A key assumption of the RR theory is the presence of a single Lyapunov length scale $L_{\rm Lyap} \sim l_B$. However, if the medium is turbulent, chaotic fluctuations will be present over a wide range of length scales. We generalized the theory for such a multiscale medium.

We begin by re-expressing the single-scale theory as follows. When two field lines are separated by a distance r smaller than l_B, their mean square separation $\langle r^2\rangle$ increases with distance l along the field line according to a Lyapunov-like scaling. However, when $r > l_B$, the increase is given by the usual diffusion law, where $\langle r^2\rangle$ increases by $\Delta r^2 \sim l_B^2$ for a parallel displacement $\Delta l \sim l_B$. Thus, we may describe the evolution in the two regimes, $\langle r^2\rangle < l_B^2$ and $\langle r^2\rangle > l_B^2$, by the following two differential equations:

$$d\langle r^2\rangle/dl \sim 2\langle r^2\rangle/l_B, \qquad d\langle r^2\rangle/dl \sim l_B^2/l_B = l_B. \tag{1.3}$$

Consider now a tangled magnetic field with a range of scales, and assume that the statistics of the magnetic field fluctuations are described by the Goldreich & Sridhar (1995, GS) theory of Alfvenic MHD turbulence. In a GS turbulent cascade, there is a range of scales $l_\perp$ perpendicular to the field, extending from a minimum scale $l_{\perp,\rm min}$ to a maximum scale l_B. The fluctuations are anisotropic, so that for a given perpendicular scale $l_\perp$ the corresponding parallel coherence scale $l_\|$ is given by

$$(l_\|/l_B) \sim (l_\perp/l_B)^\alpha, \quad l_{\perp,\rm min} < l_\perp < l_B. \tag{1.4}$$

For simplicity, we have selected the normalization such that on the outer scale l_B, the fluctuations are isotropic: $l_\perp \sim l_{||} \sim l_B$. The index α is equal to 2/3 for strong MHD turbulence (GS).

By analogy with equations (1.3), we wrote the corresponding equations for a medium with a spectrum of fluctuations (Narayan & Medvedev 2001). We then identify three regimes for the evolution of $\langle r^2 \rangle$.

First, when $r \equiv \langle r^2 \rangle^{1/2} < l_{\perp,\rm min}$, all the fluctuation scales in the medium contribute to Lyapunov-like growth (assuming they all behave chaotically), so

$$\frac{d\langle r^2\rangle}{dl} \sim 2 l_{\perp,\rm min} \int_{1/l_B}^{1/l_{\perp,\rm min}} \frac{d\ln k_\perp}{l_{||}} \sim \frac{2\langle r^2\rangle}{l_{||,\rm min}}, \tag{1.5}$$

where we have written $k_\perp = 1/l_\perp$, and we have ignored constants of order unity in the normalization of the integral. We see that the effective Lyapunov scale for the growth of r is the parallel coherence length of the *smallest* scale fluctuations in the medium, i.e.

$$r \sim r_0 \exp(l/l_{||,\rm min}), \qquad l_{||,\rm min} \sim {l_{\perp,\rm min}}^{\alpha} l_B^{1-\alpha}. \tag{1.6}$$

Since usually $l_{||,\rm min} \ll l_B$, the growth is rapid.

Once r exceeds $l_{\perp,\rm min}$, the evolution switches to a second regime. We continue to have Lyapunov-like growth from scales $l_\perp > r$, but there is diffusion-like growth for scales $l_\perp < r$. The evolution equation for ${l_{\perp,\rm min}}^2 < \langle r^2\rangle < l_B^2$ thus becomes

$$\frac{d\langle r^2\rangle}{dl} \sim 2\langle r^2\rangle \int_{1/l_B}^{1/r} \frac{d\ln k_\perp}{l_{||}} + \int_{1/r}^{1/l_{\perp,\rm min}} d\ln k_\perp \frac{l_\perp^2}{l_{||}}. \tag{1.7}$$

If $l_B \gg r \gg l_{\perp,\rm min}$, each integral in equation (1.7) is dominated by the scale r. Substituting equation (1.4) in equation (1.7), we then obtain the solution

$$r/l_B \sim (l/l_B)^{1/\alpha}. \tag{1.8}$$

Remarkably, the separation between two neighboring field lines becomes of order l_B for a parallel translation of only $\sim l_B$; this is much faster than in the RR theory which requires a parallel translation $\sim 30 l_B$. The solution $r \propto l^{1/\alpha}$ corresponds exactly to $l_\perp \propto l_{||}^{1/\alpha}$ in the turbulence model (1.4). Thus, the rms separation of field lines grows along the "Goldreich-Sridhar cone."

When $r > l_B$, we enter a third regime, which corresponds to isotropic diffusion.

From equation (1.8), it is clear that thermal conduction in a multiscale chaotic field is almost as efficient as in Spitzer's theory. Replacing L_{RR} by l_B in equation (1.2), we estimate $\kappa_{\rm turb} \sim \kappa_{\rm Sp}/3$.

We have numerically integrated the differential equations (1.5) and (1.7), starting with an initial separation $r = 10^{-13} l_B \sim \rho_e$, and assuming $\alpha = 2/3$ as appropriate for strong turbulence in the GS model. Fig. 1.2a shows four numerical solutions for the evolution of r as a function of distance l along the field line, corresponding to four choices of the minimum scale $l_{\perp,\rm min}$ of the turbulence. We see exponential growth of r for $l < l_{\perp,\rm min}$, and power-law growth for larger separations, confirming the scalings given in (1.6) and (1.8).

Let us define the decorrelation length $L_{\rm dec}$ as the distance along the field line for which the transverse separation r becomes equal to l_B. Fig. 1.2b shows how $L_{\rm dec}$ depends on $l_{\perp,\rm min}$. When $l_{\perp,\rm min} \sim l_B$, the turbulence is dominated by a single (outer) scale. This corresponds

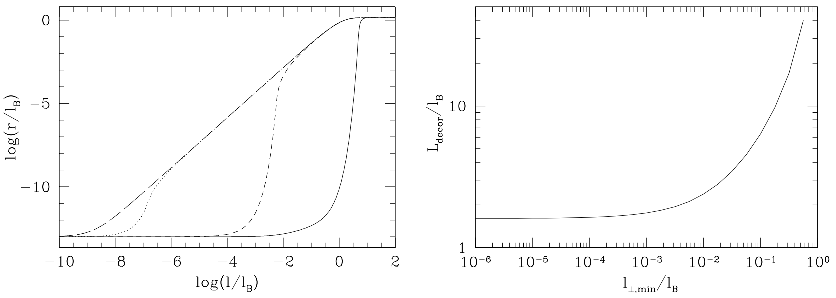

Fig. 1.2. (a) Variation of the rms transverse displacement r of two field lines vs longitudinal distance l along the field lines, for $\alpha = 2/3$ and four choices of the minimum scale of turbulence are shown. (b) Variation of the decorrelation length $L_{\rm dec}$ vs $l_{\perp,\rm min}$. Note that $L_{\rm dec} \lesssim 2l_B$ for $l_{\perp,\rm min} \lesssim 10^{-2} l_B$.

to the RR theory, and in this limit the decorrelation length is large, as expected. However, for $l_{\perp,\rm min} \lesssim 10^{-2} l_B$, we find that $L_{\rm dec}$ is quite small, asymptoting to $\sim 1.6 l_B$. Since $L_{\rm dec}$ is the analog of the Rechester-Rosenbluth length L_{RR} for a multiscale medium, we may replace L_{RR} by $L_{\rm dec}$ in equation (1.2) to estimate the diffusion constant in a turbulent medium

$$\kappa_{\rm turb} \sim (l_B/L_{\rm dec})\,\kappa_{\|} \sim \kappa_{\rm Sp}/5, \qquad t_{\rm turb} \sim 5t_{\rm Sp}. \tag{1.9}$$

Thus, if turbulence extends over at least two decades in scale, conduction is very efficient and approaches the Spitzer level to within a factor of a few. Since eqs. (1.5), (1.7) are approximate and may contain numerical coefficients of order unity multiplying the integrals, the numerical factor of 1/5 in eq. (1.9) is approximate as well and may be as high as $\sim 1/3$ which corresponds to the parallel diffusion.

1.3 Conclusions

We have shown that if the field is chaotic over a wide range of length scales (factor of 100 or more), as might happen with MHD turbulence (GS), thermal conduction is boosted to within a factor $\sim 3-5$ of the Spitzer value. Such strong conduction will have a significant effect on galaxy clusters. It can transport heat to the center of a cluster to replace the energy lost through cooling, and it can also eliminate any thermal instability in the cooling gas. Thus, it may well reduce the need for large-scale mass dropout in cooling flows.

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