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Adiabatic Growth of Massive Black Holes

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Abstract

We discuss the process of adiabatic growth of central black holes in the presence of a stationary, pre-existing distribution of collisionless stars. Within the limitations of the assumptions, the resulting models make robust physical predictions for the presence of a central cusp in the stellar and dark matter density, a Keplerian rise in the velocity dispersion, and a significant tangential polarization of the velocity tensor. New generations of numerical models confirm and extend previous results, permit the study of axisymmetric and triaxial systems, and promise new insights into the dynamics of the central regions of galaxies. These studies enable detailed comparisons with observations, further our understanding on the fueling processes for AGNs and quiescent black holes, and help elucidate the secular evolution of the inner regions and spheroids of galaxies.

1.1 Introduction

Given the premise that the massive central dark objects in normal galaxies in the local Universe are in fact supermassive black holes (Lynden-Bell 1969; Rees 1990; Kormendy & Richstone 1995; Richstone et al. 1998), we can entertain a number of conjectures about the interaction of the central black hole with its environment. Obvious questions to consider include: formation scenarios for the black hole (e.g., Rees 1984; Shapiro, this volume); the demographics of the present population of black holes (Richstone and Ho, this volume); the fueling of active nuclei (Blandford, this volume); the interaction of the active nucleus with its environment (Begelman, this volume); and, the effect of the central object upon the surrounding stellar population and the larger-scale structure of the host galaxy (Burkert, Gebhardt, Haehnelt, and Merritt, this volume). Hence, one can also test whether the inferred effects of the central object are consistent with observations, and whether additional observational constraints can be placed on either the presence or the evolutionary history of the central black hole.

A particular assumption can be made (with the caveat, that, as with all assumptions, it may be false) that a substantial increase in the mass of the central black hole takes place after initial formation, and that the mass is in some rigorous sense (to be established) added slowly to a pre-existing seed black hole. This is the assumption of *adiabatic growth*, which will be reviewed here. It leads to some nontrivial, testable predictions for the effects of black holes on their environments.

A central supermassive black hole dynamically dominates the surrounding stellar population inside some characteristic radius, $r_h = GM_\bullet/\sigma^2$, where $M_\bullet$ is the mass of the black hole and σ is the velocity dispersion of the stars outside the radius of influence. A natural "shortest" time scale for growth of a black hole is the "Salpeter" time scale, $t_S = M_\bullet/\dot{M}_{\rm Edd} \approx 5 \times 10^7$ yr, where $\dot{M}_{\rm Edd}$ is the usual Eddington accretion rate. The dynamical time scale inside r_h is just $t_{dyn} = r_h/\sigma$. For the Milky Way, $t_{dyn}(r_h) \sim 10^4$ yr, for $M_\bullet \sim 2 \times 10^6 M_\odot$ and $\sigma \approx 66\,{\rm km\,s^{-1}}$. If the observed correlation between dispersion and black hole mass holds, then $M_\bullet \propto \sigma^4$ (Gebhardt et al. 2000a; Ferrarese & Merritt 2000; Tremaine et al. 2002), and hence $t_{dyn} \propto \sigma$. Hence, we conclude that for reasonable black hole masses ($M_\bullet \lesssim 10^{10} M_\odot$), the dynamical time scale inside r_h is always much shorter than the Salpeter time scale, and therefore the likely time scale for black hole growth through accretion of baryonic matter is much longer than the dynamical time scale inside the radius at which the black hole dominates the dynamics. We thus conclude that there may be a broad range of situations under which black hole growth is "adiabatic" and the assumptions of these studies hold. The stellar population will generally form a density cusp, $\rho \propto r^{-A}$, inside r_h, with the stellar velocity dispersion showing a Keplerian rise $\sigma(r) \propto r^{-1/2}$ inside the cusp (Peebles 1972; Bahcall & Wolf 1976; Young 1980; Quinlan, Hernquist, & Sigurdsson 1995).

1.1.1 Assumptions

We consider the response of a stellar distribution function (DF) to the slow growth of a massive central black hole. The initial conditions assume that there is no central black hole to begin with (or more realistically a seed black hole with a negligible initial mass), and there are usually implicit assumptions that the mass is "magically" added to the black hole—that is to say, the mass of the central object is increased without necessarily withdrawing the mass from some explicit reservoir. The underlying assumption here is that the mass is accreting from some diffuse medium, such as cool gas, that is distributed like the stellar population but with a density much lower than that of the stellar mass density, and is replaced by some inflow that is an implicit outer boundary condition (e.g., Young 1980; Quinlan et al. 1995). This is *not* a necessary assumption; it is just a simplifying assumption. It is trivial to extend it to scenarios where the mass is explicitly withdrawn from some reservoir, with the added complication of having to specify the physical nature of the mass reservoir. In most situations modeled so far, where the mass comes from is not important; the response of the system is robust, independent of the source of the mass. The exception is if all the mass comes only from the black hole swallowing the most tightly bound stars, in which case the conclusions are somewhat different and the process effectively violates our assumptions of adiabatic growth.

An additional implicit assumption is that the black hole is *central*; that is, it is at the center of mass of the stellar system. In practice, the surrounding stellar system is discrete and the black hole mass is finite, so we expect the black hole to undergo quasi-Brownian motion away from the center (see, e.g., Chatterjee, Hernquist, & Loeb 2001). For masses of astrophysical interest, the displacement is typically much larger than the black hole Schwarzschild radius, r_S, but much smaller than r_h, and the time scale for wandering is short enough that the outer cusp is not carried with the black hole as it moves; this can lead to modification of the cusp profile at small radii ($r \lesssim 10^{-3} r_h$ for typical $M_\bullet$) as the black hole wandering produces rapid fluctuations in the central potential seen by stars in the inner cusp (see Sigurdsson 2003).

A final fundamental consideration is whether the dynamics of the stellar population are "collisionless"—that is, whether the relaxation time scale for a population of N stars, $t_R \sim N t_{dyn}/8 \ln \Lambda$, is shorter or longer than the evolutionary time scale of the stellar system, usually taken to be the Hubble time, $t_H \sim 10^{10}$ yr (e.g., Spitzer 1971; Hills 1975). The response of a relaxed stellar system to the presence of a central massive black hole has been extensively considered, primarily in the context of globular clusters, or in the context of initial black hole formation and rapid growth in protogalaxies (Bahcall & Wolf 1976, 1977; Lightman & Shapiro 1977; Cohn & Kulsrud 1978; Shapiro & Marchant 1978; Shapiro 1985; Amaro-Seoane & Spurzem 2001 and Freitag & Benz 2001 and citations therein). For supermassive black holes in normal, evolved, galaxies the relaxation time scales in the inner spheroid, but outside the black hole cusp, are generally longer than the Hubble time; inside the cusp the relaxation time may be constant, increase, or decrease with decreasing radius. For those cases where the relaxation time decreases with decreasing radius, the dynamics of the stellar population surrounding the black hole *may* undergo a transition to the fully collisional regime in the inner cusp, and the discussion in the papers cited above then becomes appropriate but is beyond the scope of this review. The mean central relaxation time can be approximated as $t_R \sim 2 \times 10^9 (\sigma/200\,\mathrm{km\,s^{-1}})^3/(\rho/10^6 M_\odot\,\mathrm{pc^{-3}})$ (Young 1980). It is not sufficient that $t_R < t_H$ for non-adiabatic growth. For such relaxed cusps the relaxation time at small radii may become shorter, and, if $t_R \lesssim t_S$ at some small radius, which may well occur for a significant fraction of galactic nuclei or proto-nuclei at some point in their evolution, then any central black hole may grow by tidal disruption of stars or by swallowing stars whole, more rapidly then the cusp can dynamically readjust its structure; in such a situation, the growth is definitely non-adiabatic.

The underlying physical assumption of the "adiabatic growth" model is that as the integrals of motion change smoothly in response to the increase in central mass, the action variables for the surrounding stellar population remain invariant (Binney & Tremaine 1987). This is to be contrasted with the opposite extreme assumption of "violent relaxation," in which the potential is assumed to fluctuate rapidly compared to the dynamical time, and the DF evolves to some final statistical equilibrium state (Lynden-Bell 1967; Stiavelli 1998). The resulting "final distribution" may then be compared with observations. It should be noted that real galaxies may not have "initial" DFs that are well represented by any of the analytic or numerical distributions assumed in these models, nor is it necessarily the case that significant increase in black hole mass ever takes place under conditions in which the adiabatic approximation holds. In particular, an implicit assumption is that a relaxed stellar population is in place as an initial condition, and that significant increase in black hole mass takes place *after* (the inner region of) the galaxy is assembled. The adiabatic models are physically distinct from *ab initio* models, where a DF including a central black hole, by design, is required to satisfy the Boltzmann equation (e.g., Huntley & Saslaw 1975; Tremaine et al. 1994). The adiabatic models are also distinct from the "orbit assembly" models used to construct kinematic models of observed galaxies (Schwarzschild 1979; Richstone & Tremaine 1984, 1988; Magorrian et al. 1998).

An interesting question is whether any of these models in some sense rigorously represents real stellar systems. Nature need not settle on the analytically or numerically derived solutions of the Boltzmann equation, out of the infinite number that exist. As found by Quinlan et al. (1995), apparently small differences in some phase-space values can lead to large changes in the averaged properties of the evolved system. We may also worry whether the different techniques for constructing stationary solutions of the Boltzmann equation representing collisionless stellar objects surrounding a central black hole are actually equivalent, or whether the different techniques produce wholly distinct families of solutions, as opposed to solutions with an overlap in properties or formally identical for some range of parameters.

1.2 Spherical Growth

The response of a spherical distribution of stars to the adiabatic growth of a central black hole in the collisionless limit was first considered by Peebles (1972) for an isothermal sphere. Young (1980) confirmed the primary result that a density cusp $\rho \propto r^{-3/2}$ would form, with an associated velocity dispersion cusp, $\sigma(r) \propto r^{-1/2}$; he also pointed out that the velocity anisotropy, $\beta(r) = 1 - \langle v_t^2 \rangle / \langle 2v_r^2 \rangle$, becomes negative (tangentially biased) at small radii, where v_t and v_r is the tangential and radial velocity, respectively. Goodman & Binney (1984) showed that when $\beta(0) = 0$ the distribution is isotropic at the center for an initial isothermal distribution (see also Binney & Petit 1989), and Lee & Goodman (1989) generalized the approximate solution of the problem to axisymmetric rotating distributions. The basic physics of the problem for a spherical system are discussed in Shapiro & Teukolsky (1983 and reference therein), as a simple application of Liouville's theorem. Their Equation 14.2.9 shows the response of a spherical system, with some initial DF $f(E)$, to a central black hole. The final density $n(R) = 4\pi \int f(E)\sqrt{[2(E-\Phi)]}dE \propto r^{-1/2} \times r^{-1}$ for $f(E) \to$ constant, appropriate for the $n = 0$ case discussed by Quinlan et al. (1995).

Quinlan et al. (1995) generalized the result to a broad range of initially spherical DFs and found that for different DFs the final cusp slope may be very different, even for near-identical initial spatial density profiles. They also found that, in contrast with the result for initially isothermal distributions, for some initial DFs the polarization of the velocity distribution is generic and always tangentially biased, and that the tangential bias may persist to zero radius. The velocity distribution is in general non-Gaussian, and initially non-Gaussian distributions may evolve to be either closer to or farther from Gaussian in response to the black hole growth (Sigurdsson, Hernquist, & Quinlan 1995). The net results are distinct, but unfortunately do not provide a simple or unique prediction for the final spherical distribution of a stellar population responding adiabatically to the growth of a central black hole. The semi-analytic results of Quinlan et al. were confirmed numerically in a companion paper by Sigurdsson et al. (1995), who extended the numerical methodology to a family of non-spherical models.

A major purpose for producing a broad range of adiabatic growth models is for comparison with observations, for example to establish robust estimators for central black hole masses from the observed surface density profiles or spectroscopically determined projected velocity dispersion profiles. In addition to the intrinsic degeneracies between the DFs and the density and dispersion profiles, we are mostly restricted to observing projected quantities, the line integrals of the light density and velocity distribution, which lead to degeneracies in the inversion to the full volume distribution (e.g., Romanowsky & Kochanek 1997 and references therein). We are further restricted to observing the dominant light-emitting population (mainly giant, sub-giant and post-AGB stars), and the mass may be distributed differently, with different stellar populations (or dark matter) having different density profiles. Still, with the use of higher moments of the velocity distribution (van der Marel & Franx 1993; Dehnen & Gerhard 1994; van der Marel 1994a, b; van der Marel et al. 1994) strong constraints can be put on the true stellar DF; by making some "natural" assumptions (e.g., the unobserved dark matter distribution is consistent with the light distribution), strong constraints can be put on the total mass of any inferred central dark object.

1.2.1 Action

In a spherical potential, we consider some initial DF f specified by the energy E and angular momentum L. The quantity f is then also a function of the actions L and $J_r = \oint v_r dr$. As the integrals change under the adiabatic growth of the black hole, the action, by assumption, remains invariant, and f evolves to remain a fixed function of the actions (see Young 1980 and Quinlan et al. 1995 for discussion).

We want to consider some initial DF with an explicitly specified form and a corresponding density profile (see Binney & Mamon 1982; Binney & Tremaine 1987). Of particular interest is the asymptotic behavior of the density profile at small radii [$\rho(r) \propto r^{-\gamma}$; as $r \to 0$] and the corresponding asymptotic behavior of $f(E)$ in the limit $E \to \Phi(0)$, which in general is some power law $f(E) \sim [E - \Phi(0)]^{-n}$. [But note that real galaxies need not be nicely monotonic power laws, even asymptotically, but may, for example, have density inversions at small radii (e.g., Peebles 1972; Lauer et al. 2002).] For an isothermal density profile the central density approaches a constant at small radii, as does the DF at the lowest energies. In general, $0 \le \gamma \le 3$, and it is useful to distinguish between models with "analytic" cores (in the nomenclature of Quinlan et al. 1995), which have a density profile $\rho(r) \approx \rho_0 + \frac{1}{2}\rho'' r^2 + \ldots$, as $r \to 0$, and non-analytic models, which do not approximate a harmonic potential at the origin. Particular examples of analytic models include a non-singular isothermal sphere, a King model, a Plummer model, or an isochrone. Non-analytic models include (1) a singular isothermal sphere (Cipollina & Bertin 1994); (2) the $\gamma = 2$ Jaffe (1983) or $\gamma = 1$ Hernquist (1990) model; (3) the generalized "gamma" models ($0 \le \gamma \le 3$; Dehnen 1993; Tremaine et al. 1994); (4) and further spherical generalizations of these models, such as those of Navarro, Frenk, & White (1997) and Zhao (1997).

It turns out that the final density cusp slope A generally depends both on the initial density slope γ and the asymptotic divergence of the DF n; the relationship among the variables is given analytically by*

$$A = \frac{3}{2} + n\left(\frac{2-\gamma}{4-\gamma}\right). \tag{1.1}$$

As illustrated in Figure 1.1, which compares the adiabatic growth of a black hole in a $\gamma = 0$ model with an isochrone model (Hénon 1960), analytic and non-analytic models with the same, or very nearly the same, initial density profiles produce qualitatively different final density profiles. More generally, the response to the adiabatic growth of a black hole produces a cusp with a slope as low as $A = 3/2$, as originally found, up to values as steep as $A = 3$, although $A = 5/2$ is probably the steepest physically sustainable slope before collisional effects in the inner cusp necessarily dominate the dynamics. Table 1.1 lists the values of some initial and final slopes (Quinlan et al. 1995); C is the final cusp slope in the limit of an initially completely tangentially biased DF. Note that the presence of a density cusp by itself is *not* a robust indicator of a central supermassive black hole; this is clearly so, since, for example, the Jaffe (1983) model or a singular isothermal sphere, with no central black holes, have $\gamma = 2$ cusps, steeper than the $A = 3/2$ cusps predicted for the response of

* For the full derivation, see Appendix A of Quinlan et al. (1995), Gondolo & Silk (1999), and Ullio, Zhao, & Kamionkowski (2001).

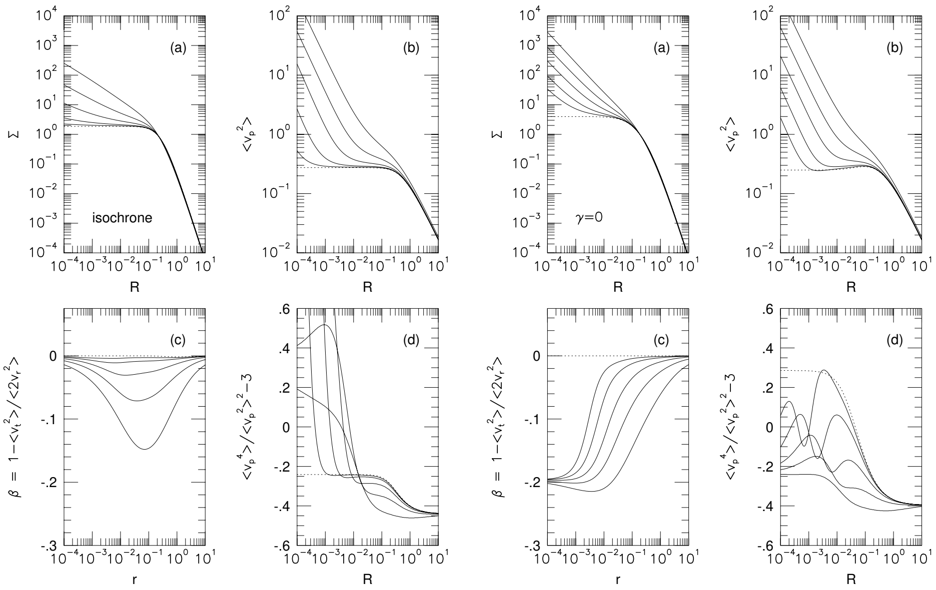

Fig. 1.1. Results of the growth of black holes of different masses, $M_\bullet = 10^{-3} - 10^{-1}$ of the total (spheroid) galaxy mass, in an isochrone (*left*) and $\gamma = 0$ (*right*) model, respectively. The dotted lines show the initial (near-identical) models. The panels show (*a*) surface density, (*b*) projected velocity dispersion, (*c*) projected anisotropy $\beta(r)$, and (*d*) kurtosis $\kappa - 3$. Note the very different final profiles despite near-identical observable initial conditions. (From Quinlan et al. 1995.)

Table 1.1. *Adiabatic density cusps*

Model	γ	n	A	C
isochrone	0	0	$3/2$	$9/4$
$\gamma = 0$	0	1	2	$9/4$
$\gamma = 1$	1	$5/2$	$7/3$	$7/3$
$\gamma = 3/2$	$3/2$	$9/2$	$12/5$	$12/5$
$\gamma = 2$	2	–	$5/2$	$5/2$

a non-singular isothermal sphere to a central black hole. On the other hand, the presence of a density cusp, a Keplerian rise in the velocity dispersion, and the kinematic signature of tangential anisotropy at small radii *are* robust indicators of a central supermassive black hole.

This does not preclude the possibility that in the absence of a central black hole actual stellar systems tend toward flat, constant density cores, whether through formation or relaxation, and that in practice cusps are in fact signatures of central black holes. We know that for a broad range of formation scenarios, stellar cusps form around central black holes (with cusps as shallow as $A = 1/2$ or as steep as $A = 5/2$); however, it is possible that in some situations binary black holes completely destroy cusps, leaving density inversions (e.g., Peebles 1972), and we know it is possible for cuspy stellar systems to exist in the absence of central black holes. Assuming that cusps are tracers of black holes, van der Marel (1999) has explored the use of the adiabatic growth models in matching observed density profiles.

1.2.2 Anisotropy

Quinlan et al. (1995) also experimented with spherical, radially anisotropic distributions (Osipkov 1979; Tonry 1983; Merritt 1985; Dejonghe 1987; Cudderford 1991; Gerhard 1993), but found it impossible to generate physical distributions with significant radial anisotropy persisting to zero radius. The general conclusion is therefore that adiabatic growth induces tangential bias at small radii, and that an initial tangential bias can, but does not necessarily, lead to steeper final cusps, compared to the equivalent isotropic model. The Keplerian velocity cusp is a robust prediction of spherical adiabatic growth models. As noted by Duncan & Wheeler (1980), however, a strong radial velocity anisotropy can mimic a Keplerian rise in velocity in projection, although there are severe concerns about the stability of any such model (Merritt 1987; Palmer & Papaloizou 1988). A robust prediction of a tangential bias induced by a central black hole is therefore potentially important, although the anisotropy is not a directly observable quantity but must be inferred from the projected moments of the velocity distribution.

As shown in Figure 1.1, the final anisotropy, $\beta(r)$, may be either zero at the black hole or remain negative at small radii. The deviation from Gaussianity is conveniently measured by the kurtosis, κ (by construction, the skew is zero for these models), or equivalently, the fourth Gauss-Hermite moment, $h_4 \approx (\kappa-3)/8\sqrt{6}$, for $h_4 \lesssim 0.03$ (van der Marel & Franx 1993; Dehnen & Gerhard 1994; Quinlan et al. 1995).

With $A = 3/2+p > 3/2$, the relaxation time scale at small radii decreases as $t_R \propto r^p$. The cusps induced in isotropic, analytic models have $p = 0$ and constant t_R; more generally, $p > 0$ and t_R can be small close to the black hole. Very close to the black hole, relaxation and collision time scales get short for strong cusps, and strong collisional effects may lead to rapid growth of the black hole, with corresponding associated depletion of the stellar population. This is certainly the case for cusps as steep as $A = 3$, and may even be a problem for shallower cusps (Frank & Rees 1976; Quinlan & Shapiro 1990; Quinlan et al. 1995; Sigurdsson & Rees 1997; Freitag & Benz 2001).

1.2.3 Non-adiabatic Growth

Formally, the adiabatic growth model implies an infinitely long time scale for accretion. In practice, of course, any growth in mass occurs on a finite time scale. We can investigate the nature of non-adiabatic growth without losing the predictive power of the adiabatic models.

Sigurdsson et al. (1995; see also Hernquist & Ostriker 1992; Hernquist, Sigurdsson, & Bryan 1995; Sigurdsson et al. 1997a) explored the time scale for adding mass, and concluded that, for time scales $t \gtrsim 10\,t_{dyn}(r_h)$, the adiabatic approximation was satisfied for the resolution of the models. The use of N-body modeling also showed the final distributions after adiabatic growth were stable; stability is not guaranteed by the adiabatic growth process, nor is there a general analytic criterion for stability of arbitrary DF.

Adiabatic growth formally also implies reversibility. Sigurdsson & Hernquist (unpublished) experimented with numerical models in which a central black hole grown adiabatically in a spherical stellar distribution was *removed* adiabatically. The original distribution was in fact recovered to within the resolution of the models.

In general, violent formation can lead to either galaxies with constant-density cores (e.g., Lynden-Bell 1967; van Albada 1982; Norman, May, & van Albada 1985; Burkert, this volume) or singular profiles (e.g., Aarseth 1966; Fillmore & Goldreich 1984; Bertschinger 1985; Navarro et al. 1997).

Stiavelli (1998) and Ullio et al. (2001) explored non-adiabatic growth with a pre-existing black hole and found results that did not deviate strongly from the case of adiabatic growth. More recently, MacMillan & Henriksen (2002) suggested that non-adiabatic accretion of dark matter might account for the $M_\bullet - \sigma$ relation, which is not explained by a simple adiabatic compression of the dark matter halo (Dubinski & Carlberg 1991). Adiabatic compression of the dark matter in the inner regions by the formation of a central black hole is potentially interesting, as it can lead to increased rates of dark matter accretion onto the black hole, and to higher rates of dark matter self-interaction, for models in which such interactions may occur (Gondolo & Silk 1999; Ostriker 2000).

Sigurdsson et al. (1995) also found that steep initial density cusps were vulnerable to violent disruption by the "wandering" of the central black hole. Black hole mergers will also efficiently destroy steep stellar cusps around a black hole (Makino & Ebisuzaki 1996; Quinlan & Hernquist 1997; Faber et al. 1997; Milosavljević & Merritt 2001; Zier & Biermann 2001; Hemsendorf, Sigurdsson, & Spurzem 2002; Ravindranath, Ho, & Filippenko 2002).

1.3 Non-spherical Systems

The obvious next approximation beyond spherical (isotropic and anisotropic) models is to consider axisymmetric ones. A number of families of two- and three-integral axisymmetric models exist in the literature (e.g., Evans 1993; Hunter & Qian 1993; Kuijken & Dubinski 1994; Qian et al. 1995; Gebhardt et al. 2000b; Lynden-Bell 2002).

Van der Marel et al. (1997a) constructed a detailed model for the central black hole in M32, and van der Marel, Sigurdsson, & Hernquist (1997b) ran a numerical model, using techniques developed for adiabatic growth simulations, to demonstrate its stability. A concern remains that such models may be unstable to $m = 1$ modes, which are typically suppressed in numerical simulations (if not, they can arise spontaneously through numerical artifacts, which make it difficult in general to identify physical instabilities). Lee & Goodman (1989) modeled adiabatic growth of black holes in approximate rotating, isothermal axisymmetric models. Rather interestingly, they found that the rotation curve rises more rapidly than the dispersion curve, but not enough to account for the observed high rotation in the inner regions of some systems. A more general exploration of adiabatic growth in non-rotating axisymmetric models was done by Sigurdsson & Hernquist (unpublished, see Fig. 1.2). They found that axisymmetric models are quite similar to spherical models, particularly in that the tangential anisotropy is induced in the cusp and the density profile becomes rounder at small radii (Fig. 1.2). Leeuwin & Athanassoula (2000) simulated adiabatic growth in Lynden-Bell (1962) models and obtained results consistent with those of Lee & Goodman (1989), including rounding of the inner density profile and a significant rise in the rotation velocity inside the cusp, consistent with the tangential polarization of the central black hole.

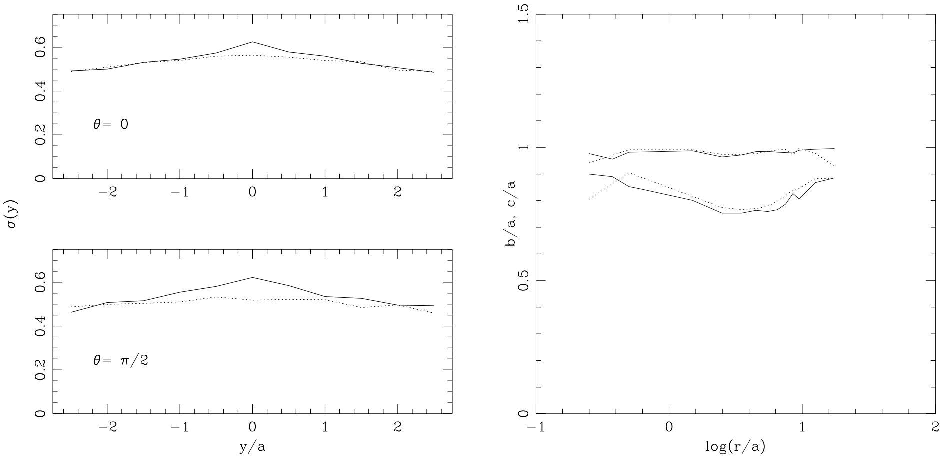

Fig. 1.2. Results of the growth of a black hole in an "Evans model." The panels show the projected density and axis ratios for edge-on and face-on inclinations before (*dotted line*) and after (*solid line*) black hole growth. The inner density profile becomes rounder in response to the black hole, and the characteristic Keplerian dispersion profile is seen in projection. (From Sigurdsson & Hernquist, unpublished.)

1.3.1 Triaxial Systems

Real galaxies are generally not spherical or axisymmetric, but triaxial (Binney 1976; Franx, Illingworth, & de Zeeuw 1991; Ryden 1992, 1996; Tremblay & Merritt 1995; Bak & Statler 2000). We expect triaxial galaxies to form from general cosmological initial conditions (e.g., Norman et al. 1985; Dubinski & Carlberg 1991) and from galaxy mergers (Barnes 1988, 1992; Hernquist 1992, 1993). Exact analytic models exist for triaxial galaxies with cores (Schwarzschild 1979; de Zeeuw 1985; Statler 1987; van de Ven et al. 2003). Observationally, we also see that the density profiles of the spheroidal component of galaxies generally continue to rise toward the center, with $0.5 \lesssim \gamma \lesssim 2.3$ (e.g., Lauer et al. 1995; Gebhardt et al. 1996; Faber et al. 1997; Ravindranath et al. 2001). There are also observed correlations between the cusp slope γ and the global properties of the galaxy, including shape in the form of boxy or disky isophotes (e.g., Faber et al. 1997).

Historically, dynamical arguments suggest that the presence of a strong central cusp ($\gamma > 1$) induces chaos in the orbit families that populate the galaxy, driving the system away from strong triaxiality (e.g., Gerhard & Binney 1985; Norman et al. 1985; Merritt & Valluri 1996; Merritt & Quinlan 1998; Merritt 1997, 1999). The argument is that central cusps or central point masses scatter the box orbits that support triaxiality in galaxies, inducing a population of chaotic orbits that drive figure evolution toward axisymmetry (Miralda-Escudé & Schwarzschild 1989; Lees & Schwarzschild 1992; Fridman & Merritt 1997; Valluri & Merritt 1998). Hence, central supermassive black holes should preclude the presence of triaxiality at small radii, and might drive global figure evolution of the system (Norman et al. 1985; Merritt & Quinlan 1998). This is potentially very important because triaxial potentials support fueling of the central black hole through material falling into it on box orbits (e.g., Norman & Silk 1983; Valluri & Merritt 1998) or by gas traveling on intersecting orbits that drive dissipation and inflow, thus providing a direct link between the dynamics in the center of the galaxy and its global properties. In the extreme case of disk systems, analogous instabilities exist (e.g., Hasan & Norman 1990; Sellwood & Valluri 1997).

1.3.2 Adiabatic Growth and Triaxiality

Holley-Bockelmann et al. (2001; see also Sigurdsson et al. 1997b, 1998) showed that applying numerical adiabatic growth techniques to "squeeze" an initially spherically symmetric cuspy DF could produce a stable, stationary, cuspy triaxial configuration with well-characterized phase-space properties. A key aspect of the models is that they contain a central cusp of near-constant slope and near-constant axis ratios, with significant triaxiality at all radii resolved by the models (Holley-Bockelmann et al. 2001, 2002). Galaxies with density cusps support different stellar orbits than, for instance, $\gamma = 0$ core models (Gerhard & Binney 1985; Gerhard 1986; Pfenniger & de Zeeuw 1989; Schwarzschild 1993; de Zeeuw 1995; Merritt 1999; Holley-Bockelmann et al. 2001). The set of models thus produced provide a starting point for investigation of the adiabatic growth of central black holes in cuspy, triaxial potentials. A black hole is then grown using the previously developed numerical N-body techniques (Sigurdsson et al. 1995; Holley-Bockelmann et al. 2002).

Following Holley-Bockelmann et al. (2001, 2002), consider a black hole grown in a triaxial Hernquist model with initial cusp slope $\gamma = 1$. As the black hole grows, both the cusp slope γ and central velocity dispersion σ_p increase, as in spherical and axisymmetric models. The cusp settles to an equilibrium value $\gamma \simeq 2.05$, measured at projected ellipsoidal radius $Q = 10^{-1.3}$, with projected central dispersion $\sigma_p \simeq 0.7$, measured at projected ellipsoidal radius $Q = 10^{-2.3}$. These results are characteristic of adiabatic black hole growth in cuspy galaxies and can be compared both to analytic estimates for adiabatic black hole growth in a spherical $\gamma = 1.0$ model, which predict $\gamma = 7/3$ and $\sigma_p = 0.75$ (Quinlan et al. 1995), and to the results from N-body simulations where $\gamma \approx 2.2$ and $\sigma_p \approx 0.65$ (Sigurdsson et al. 1995). The fact that the measured cusp slope is less than the analytic value is to be expected, since the cusp slope is measured over a finite radial range near the center, and it is not the asymptotic $q = 0$ value.

As the black hole grows, the inner regions become rounder (Fig. 1.3*b*); the central 10% of the mass, corresponding to an ellipsoidal radius $q = \sqrt{x^2 + (y/b)^2 + (z/c)^2} < 0.1$, is close to spherical with axis ratios $a : b : c = 1.0 : 0.95 : 0.92$. The shape evolution in the outer regions is much less dramatic. Following the growth of the black hole, the model exhibits a marked shape gradient, becoming more strongly triaxial with increasing radius. Despite the nearly axisymmetric shape at the center, the inner region is still triaxial enough to influence the stellar-orbital dynamics (Statler 1987; Hunter & de Zeeuw 1992; Arnold, de Zeeuw, & Hunter 1994).

The final state of this model features several hallmarks of a black hole-embedded triaxial figure. Figure 1.3 shows the properties of this object as a function of ellipsoidal radius q at $T = 40$ ($12.8\, t_{dyn}$ at $q = 1$), well after the model black hole has stopped growing. Figure 1.3*a* shows the $\gamma \approx 2$ density cusp induced by the black hole inside $\log q = -1$. At a larger radii $\log q > -1$, however, this plot demonstrates that the system retains the original Hernquist density profile. Figure 1.3*b* shows explicitly the strong shape gradient in the model. Inside r_h, both the projected and intrinsic velocity dispersions exhibit a strong central cusp (panels *c* and *d*). In the outskirts, where the model maintains its triaxiality, the projected velocity distributions follow $\sigma_x > \sigma_y > \sigma_z$, in accord with a triaxial model where $a > b > c$. How-

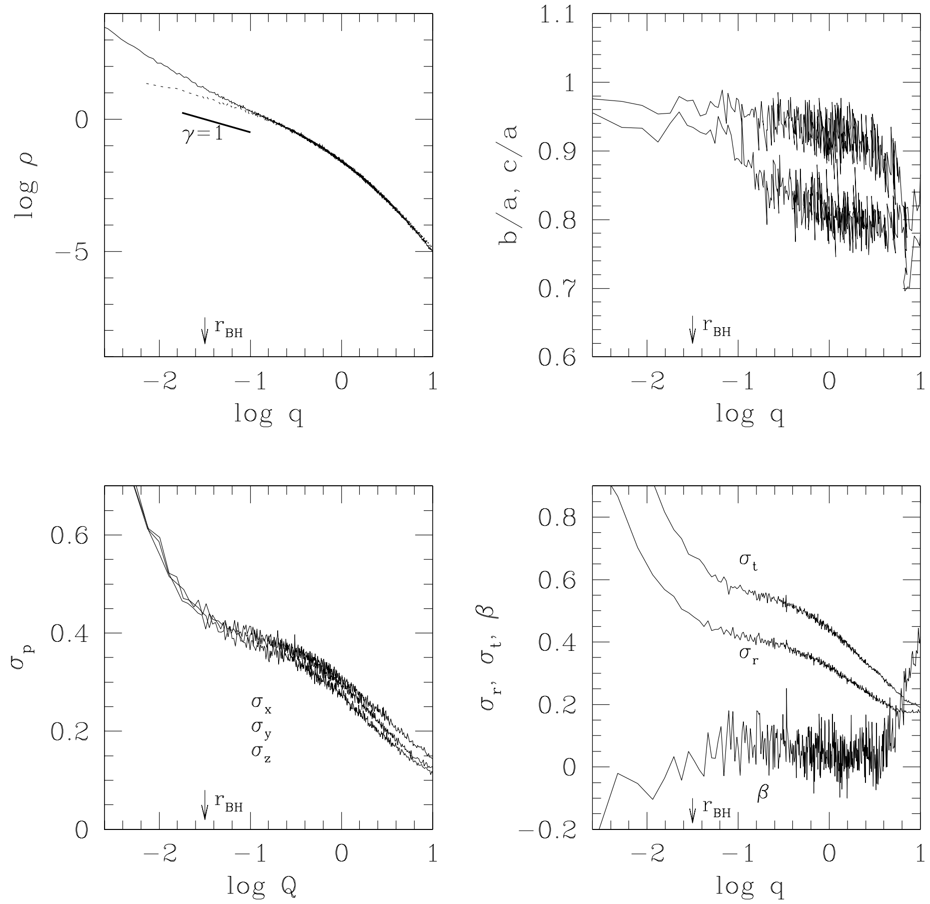

Fig. 1.3. The structural and kinematic properties of a cuspy triaxial galaxy model with a massive central black hole. *Upper left*: density profile; *upper right*: intermediate and minor axis lengths as a function of ellipsoidal radius; *lower left*: projected velocity dispersion along the fundamental axes, as a function of projected ellipsoidal radius; *lower right*: true radial and tangential velocity dispersion, and velocity anisotropy parameter, as a function of ellipsoidal radius. (From Holley-Bockelmann et al. 2002.)

ever, inside the cusp the projected velocity dispersions are commensurate. Interestingly, the anisotropy parameter, $\beta = 1 - \langle v_t^2 \rangle / \langle 2v_r^2 \rangle$, becomes negative near the black hole. This is consistent with models of stellar orbits around a black hole that is adiabatically grown, where $\beta = -0.3$ (Goodman & Binney 1983; Quinlan et al. 1995). Exterior to the black hole's radius of influence, the system is radially anisotropic ($\beta > 0$), as expected for a triaxial galaxy.

Poon & Merritt (2001, 2002), using Schwazschild's orbit-assembly technique, have now also found models with triaxiality at small radii in the presence of a central black hole and a surrounding cusp.

Clearly, it is possible for some significant triaxiality to persist both in the presence of a central cusp, and, more importantly, in the presence of a central black hole.

1.3.3 Chaos

If a significant fraction of the orbits in triaxial models containing central black holes become chaotic, then by the ergodic theorem the shape of the distribution must evolve toward sphericity (possibly halting when axisymmetry is reached). It is clear that the onset of chaos in the most tightly bound orbit families leads to a rapid change in the inner structure of the model galaxies. In the outer regions, orbits stay regular even after repeated passages near the potential center. It is possible that many of these orbits are actually chaotic orbits that are "sticky" (Siopis & Kandrup 2000), with a very long diffusion time scale. The course grainedness of the numerical model potential seems to argue against this explanation; a course-grained potential effectively creates holes in the Arnold web (Arnold 1964) through which an otherwise confined orbit may escape. Figure 1.4 illustrates what is probably happening in these models; strong scattering is taking place, inducing some chaos, but triaxiality is sustained by the persistence of resonant boxlet orbits that scatter between each other, rather than into true chaotic orbits.

There are two important issues to be explored here.

- When a spherical (or axisymmetric) model is squeezed adiabatically into a triaxial configuration, one of the implicit assumptions of adiabatic growth is violated. The evolution of the potential has discretely broken a symmetry underlying the second and third integrals of motion; incidentally, the reversibility of the process is also destroyed. However, the action is still conserved, at least approximately, so the DF must bifurcate, leaving an excluded region of phase space. In 2-D this region would be forbidden; in 3-D other bifurcating branches can cross-over into the newly created vacant region of phase space, but do not in general fill it. By construction, this technique leaves vacant islands in phase space, and consequently we reach stable and stationary triaxial configurations despite the presence of the black hole. Numerically these are robust solutions, but they are not guaranteed to be robust physically. It is possible that small amounts of relaxation or potential fluctuation could rapidly refill these vacated phase-space regions, leading to boxlet–chaos transitions, breaking the dynamical equilibrium constructed for boxlet–boxlet transitions. Some of this is seen through chaos induced by numerical scattering in the models.

  We cannot yet be sure that our triaxial adiabatic solutions are physically robust ones achieved by natural systems, as distinct from mathematical curiosities; nevertheless, they are potentially very interesting solutions for triaxial systems.
- We also do not understand well how a transition to chaos occurs in these systems. The scattering by the central singular potential, in and of itself, need not induce chaos. After all, perturbed orbits in Keplerian potentials are regular. Some insight can be gained by considering a toy 2-D model (to be compared with the dynamics in the principal plane of a triaxial system).

  Following Devaney (1982), consider a homogeneous, *anisotropic* potential of degree k, $\Phi(r,\psi) \propto r^{-k}$. There are three special values of $k = \{0, 1, 2\}$, the first two corresponding to the isothermal and Keplerian potential, respectively. We consider the characteristic exponents in the linearized theory for 2-D orbits and explore the effects of varying k and the degree of anisotropy. Solving for the characteristic exponent λ, we find

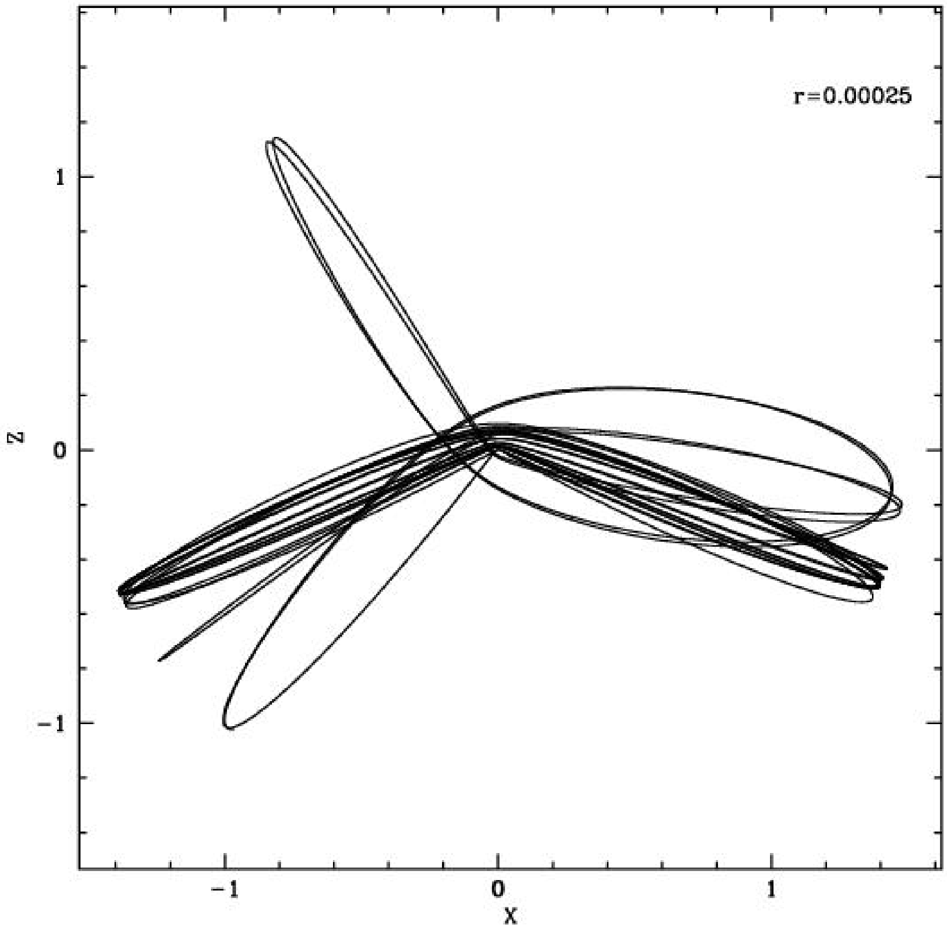

Fig. 1.4. The evolution of a banana boxlet orbit due to scattering after close approach to a central black hole in a triaxial potential. The banana orbit flips to a fish orbit, another resonant boxlet, and does not become chaotic due to strong scattering. (From Holley-Bockelmann et al. , in preparation.)

$$\lambda(k,r) = \frac{1}{2}[(\frac{1}{2}k-1)v_r - [(\frac{1}{2}k-1)^2 v_r^2 - 4\Phi''(\psi)]^{1/2}] \tag{1.2}$$

where Φ'' is the second angular derivative of the potential, which is by hypothesis self-similar (i.e., the shape of the isopotential contours is independent of radius). Clearly, realistic galaxy models are not homogeneous, but at large and small radii they well approximate a homogeneous potential, and the dynamics, in particular the orbit divergences, are due to *local* potential gradients.

Instability formally occurs when λ is imaginary, so the critical points occur when $[(\frac{1}{2}k-1)^2 v_r^2 - 4\Phi''(\psi)] = 0$. Using Poisson's equation, $\Phi'' = 4\pi r^2\rho - 2v_c^2 - k(k-1)\Phi$, where, by hypothesis, $\Phi''(\psi)$ is independent of radius. We therefore conclude, that for Keplerian and isothermal potentials orbits in this model go chaotic at large r, not crossing through the center. For $k = 2$, appropriate for the outer regions of galaxy models, orbits that are chaotic anywhere are chaotic everywhere.

If the dynamics of the toy 2-D homogeneous model are a good indicator, then the onset of chaos in triaxial models occurs not at small radii, but in the transition region between the inner cusp and the region outside r_h.

1.3.4 *Uses of Triaxial Models*

Much work remains to be done. The following are some implications worth exploring.

- The gas dynamics on intermediate scales and possibilities for AGN fueling may depend strongly on even mild triaxialities on small scales.
- The dynamical evolution of merging dwarf galaxies and the fate of low-mass black holes merging with massive galaxies may be sensitive to triaxiality persisting as the central black hole becomes more massive. Dynamical friction processes may be expedited through centrophilic orbits in triaxial potentials.
- On small scales, triaxiality inside r_h may promote rapid loss-cone refilling and be critical for sustaining high tidal-disruption rates and the influx of low-mass compact objects coalescing with the central massive black hole (Sigurdsson 2003).

1.4 Conclusions

Adiabatic growth models provide valuable physical insights into the dynamical processes and interactions of central massive black holes with their surroundings. Over the last 30 years, a broad range of physically robust results on the dynamical influence of the black hole on the surrounding stellar population have contributed to our confidence in the reality of supermassive black holes and helped provide strong quantitative constraints on black hole masses. Additional physical insight has been gained in understanding the ever-fascinating subtleties of Newtonian dynamics of many-body systems. New generations of N-body models will allow a broader exploration of more realistic, less symmetric systems, more tests of stability and secular evolution, and possibly a deeper understanding of dynamical processes on small and large scales.

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