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Formation of Massive Black Holes in Dense Star Clusters

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Abstract

We review possible dynamical formation processes for central massive black holes in dense star clusters. We focus on the early dynamical evolution of young clusters containing a few thousand to a few million stars. One natural formation path for a central seed black hole in these systems involves the development of the Spitzer instability, through which the most massive stars can drive the cluster to core collapse in a very short time. The sudden increase in the core density then leads to a runaway collision process and the formation of a very massive merger remnant, which must then collapse to a black hole. Alternatively, if the most massive stars end their lives before core collapse, a central cluster of stellar-mass black holes is formed. This cluster will likely evaporate before reaching the highly relativistic state necessary to drive a runaway merger process through gravitational radiation, thereby avoiding the formation of a central massive black hole. We summarize the conditions under which these different paths will be followed, and present the results of recent numerical simulations demonstrating the process of rapid core collapse and runaway collisions between massive stars.

1.1 Introduction

The main focus of this chapter is on the formation of massive black holes (BHs) through stellar-dynamical processes in young star clusters. Here by "massive" we mean BHs with masses in the range $\sim 10^2 - 10^4\,M_\odot$, which could be "intermediate mass" BHs in small systems such as globular clusters (van der Marel, this volume) or "seed" BHs in larger systems such as proto-galactic nuclei. The later growth of seed BHs by gas accretion or stellar captures to form supermassive BHs in galactic nuclei is discussed by Blandford (this volume). The early dynamical evolution of dense star clusters on a time scale $t \lesssim 10^7$ yr is completely dominated by the most massive stars that were formed in the cluster. This early phase of the evolution is therefore very sensitive to the stellar initial mass function (IMF), particularly at the high-mass end (see Clarke, this volume). One possibility, which we will discuss in some detail here, is that the massive stars could drive the cluster to core collapse before evolving and undergoing supernova explosions. Successive collisions and mergers of these massive stars during core collapse can then lead to a runaway process and the rapid formation of a very massive object containing the entire mass of the collapsing cluster core. Although the fate of such a massive merger remnant is rather uncertain, direct "monolithic" collapse to a BH with little or no mass loss is a likely outcome, at least for sufficiently low metallicities (Heger et al. 2003).

Alternatively, if the most massive stars in the system evolve and produce supernovae before the onset of this runaway collision process, the collapse of the cluster core will be reversed by the sudden mass loss, and a cluster of stellar-mass BHs will be formed. The final fate of this cluster is also rather uncertain. For small systems such as globular clusters, complete evaporation is likely (with all the stellar-mass BHs ejected from the cluster through 3-body and 4-body interactions in the dense core). This is expected theoretically on the basis of simple qualitative arguments (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993) and has been demonstrated recently by direct N-body simulations (Portegies Zwart & McMillan 2000). However, for larger systems such as proto-galactic nuclei, contraction of the cluster to a highly relativistic state could again lead to successive mergers (driven by gravitational radiation) and the formation of a single massive BH (Quinlan & Shapiro 1989).

Because of the great complexity and variety of dynamical processes involved, questions related to BH formation in dense star clusters are best studied using numerical simulations. On today's computers, this can be done with direct N-body simulations for $N \approx 10^3 - 10^5$ stars (see, e.g., Aarseth 1999) or with Monte Carlo (MC) techniques (see Sec. 1.2) for up to $N \approx 10^7$ stars. Observationally, this large range covers a variety of well-studied young star clusters, from "young populous clusters" (e.g., Arches, R136) to "super star clusters" (Whitmore 2003). However, realistic, star-by-star simulations of larger systems, on the scales of entire galactic nuclei, are not yet possible. For these systems, one must rely on more qualitative analyses based on extrapolations from numerical results for smaller N.

The role of stellar collisions in dense galactic nuclei was first consider to explain the quasar/active galactic nucleus phenomenon (e.g., Spitzer & Saslaw 1966). Colgate (1967) pointed out that, when collisions first set in a stellar cluster evolving from "reasonable" initial conditions, they are unlikely to be disruptive so that runaway mergers should lead to the formation of massive stars. He estimated that growth would saturate at $\sim 50\,M_\odot$ because small stars could fly across a massive star without being stopped. In "particle-in-a-box"-type simulations using a semi-analytical model for the outcome of individual collisions, Sanders (1970) found clear runaway growth up to a few hundred $M_\odot$. No proper account of the stellar dynamics was included, however; the cluster was treated as a homogeneous sphere of constant mass (the gas ejected in collisions being recycled into stars) contracting through collisional energy loss.

Stellar dynamics must be playing an important role, however, especially for massive stars, which can be affected significantly by mass segregation. Indeed, massive stars of mass m undergo mass segregation and concentrate into the cluster core on a time scale $t_{\rm segr} \simeq (\langle m\rangle/m)t_{\rm rh}$, where $\langle m\rangle$ is the average stellar mass and the overall relaxation time (at the half-mass radius $r_{\rm h}$) for a cluster of total mass M is given by

$$t_{\rm rh} \simeq 10^8\,{\rm yr}\left(\frac{r_{\rm h}}{1\,{\rm pc}}\right)^{3/2}\left(\frac{M}{10^6\,M_\odot}\right)^{1/2}\left(\frac{\langle m\rangle}{1\,M_\odot}\right)^{-1}.$$

It is clear that the most massive stars in a dense cluster can undergo mass segregation on a time scale much shorter then their stellar evolution time. When they eventually dominate the density in the cluster core, these massive stars will then *decouple dynamically* from the rest of the cluster (go out of thermal equilibrium, evolving *away* from energy equipartition) and evolve very quickly to core collapse. This process is often referred to as Spitzer's "mass-segregation instability."

The possibility of a "mass-segregation instability" in simple two-component systems (clusters containing only two kinds of stars, one much more massive than the other) was first predicted by Spitzer (1969), and the first dynamical simulations revealing mass segregation at work were performed in the 70's (Spitzer & Hart 1971; Spitzer & Shull 1975). The physics of this instability is now very well understood theoretically (Watters, Joshi, & Rasio 2000). In a remarkably prescient paper, Vishniac (1978) showed correctly for the first time that this instability must affect the early dynamical evolution of any star cluster born with a reasonable "Salpeter-like" IMF. He concluded that, as a result, "most globular clusters may undergo core collapse at an early time in the evolution of the universe. This is clearly significant as a possible mechanism for creating collapsed bodies in the center of globular clusters."* In their classic paper the same year, Begelman & Rees (1978) also mentioned the combination of mass segregation and runaway merging as "one of the quickest routes to the formation of a massive object in a dense stellar system."

However, the first detailed dynamical simulations of dense star clusters including stellar collisions and mergers considered clusters where all stars are initially identical (Lee 1987; Quinlan & Shapiro 1990, hereafter QS90). These Fokker-Planck (FP) simulations also included the dynamical formation of binaries through 3-body interactions and their subsequent hardening (and ejection) as a central source of energy capable of reversing core collapse and turning off collisions in clusters with a relatively low number of stars. Furthermore, in these simulations, collisions themselves can stop collapse when collisionally produced massive stars lose mass in a supernova explosion at the end of their life. The results of these early simulations suggested that runaway collisions would occur provided that $t_{\rm rh} < 10^8\,$yr (to beat stellar evolution) and $N \geq 3 \times 10^6$ (to avoid binary heating). QS90 stressed that, as a result of mass segregation, the rise in the central velocity during collapse is only moderate and collisions do not become disruptive. Although not very realistic, these early studies made plausible the idea that successive collisions and mergers of main-sequence stars could lead to the formation of a $\sim 10^2 - 10^3\,M_\odot$ object.

More recently, through direct N-body simulations of clusters containing 2000 to 65,000 stars, Portegies Zwart & McMillan (2002) showed that, in such low-N systems, dynamically

* Historical note contributed by G. Burbidge: "When we first were trying to understand what was responsible for violent events, originally in radio sources, in the early 1960s we (Hoyle, Fowler and the Burbidges') discussed the problem of gravitational collapse of massive objects (superstars) after Hoyle and Fowler had argued that the mechanism I had proposed in 1961, chain reactions of supernovae, would not work. In this early period Hoyle and Fowler wrote several papers on the collapse, and in 1964 HFB^2 published a paper in ApJ discussing all of the ramifications. We did not know how to form the beast — the work of Colgate and Spitzer showed how it would work if the star density was high enough, but the densities required were far greater than any that could reasonably exist in the centers of galaxies. But we were convinced that such phenomena must exist in the centers, and Hoyle was quite angry when much later in 1969 Lynden-Bell was given credit for the idea after a press conference. By then Hoyle and I had become convinced that the usual mechanism of accretion would not give enough energy because the efficiency of the process in terms of what *we see* must be very low, and much below 10%. Thus we turned to new and fundamental ideas concerning creation in galaxy centers which can only take place in regions of very strong gravitational fields, i.e., next to classical BHs. This is my position today. Fred and I worked on it until his death. When Vishniac's paper appeared in 1978, Fred and I discussed it and realized that mass segregation might allow a BH to form rapidly. But this was in the context of total masses of $10^5 - 10^6\,M_\odot$ — globular clusters — much less than is required for massive BHs in nuclei. Still we talked about doing more work on it. It was not pursued, probably because we were neither in a position to do it ourselves; I was just moving to Kitt Peak as director, and Fred had left Cambridge and was living up in Cumberland. But we both thought it was an important step forward."

formed binaries, far from preventing collisions (by heating the cluster and reversing collapse), actually *encourage* them by increasing the effective cross section. In these small systems, once the few massive stars have segregated to the center, one of them will repeatedly form a binary with another star and later collide with its companion when an interaction with a third star increases the binary's eccentricity. The growth of this star is ultimately stopped by stellar evolution, or by the dissolution of the cluster in the tidal field of the parent galaxy. Given the small number of stars in these simulations, the maximum mass of the collision product is only $\sim 200\,M_\odot$ when mass loss from stellar winds is negligible. The "pistol star" in the Galactic Center Quintuplet cluster (Figer et al. 1998) may provide an example of a directly observable runaway collision product of this type in a small, young star cluster.

Work in progress by the authors is now attempting for the first time to study numerically these processes for much larger star clusters, containing up to $N \approx 10^7$ stars, and resolving in detail the core collapse and runaway collisions. Section 1.2 provides a summary of the numerical methods that we are using, based on a MC technique for collisional stellar dynamics. Section 1.3 presents a few of our initial results. Our main conclusions can be summarized as follows.

Our numerical simulations show that, in the absence of stellar evolution, the core collapse time in a dense star cluster is always given by

$$t_{\rm cc} \simeq 0.1\,t_{\rm rh},$$

for any "reasonable" IMF (i.e., not too different from a Salpeter IMF) and initial cluster structure (basically, any initial density profile that is "not too centrally concentrated"). If we assume that core collapse corresponds to the onset of runaway collisions, then the condition for a runaway to occur can be written very simply

$$t_{\rm cc} \simeq 0.1\,t_{\rm rh} < \tau_*(m_{\rm max}),$$

where $\tau_*(m_{\rm max})$ is the lifetime of the most massive stars, of mass $m_{\rm max}$. From current stellar evolution calculations (e.g., Schaller et al. 1992) we know that

$$\tau_*(m_{\rm max}) \simeq 3\,{\rm Myr} \qquad {\rm for}\ \ m_{\rm max} > 30\,M_\odot,$$

i.e., nearly *constant* (this is simply because these massive stars are nearly Eddington-limited*), and also nearly independent of metallicity and rotation (within $\sim 10\%$). Therefore, under very general conditions, the simple criterion for a runaway process can be written

$$t_{\rm rh} \lesssim 3\times 10^7\,{\rm yr}.$$

This is in perfect agreement with the results of the direct N-body simulations by Portegies Zwart & McMillan (2002), although they cannot resolve the core collapse in their simulations, and instead *define* core collapse to be the onset of collisions†.

Perhaps the most interesting result from our new simulations is that the central (BH) mass produced by this runaway process may well be determined largely by the Spitzer instability: the total mass in massive stars going into core collapse will be the final BH mass, at least

* If the mass segregation time scale $t_{\rm s} \propto 1/m$ is compared to the "usual" $\tau_* \propto 1/m^3$, one could conclude erroneously that massive stars never play a role in core collapse (Applegate 1986). However, the approximate $1/m^3$ scaling of the stellar lifetime applies only for $m \lesssim 10\,M_\odot$.

† For any runaway collision process to occur, the IMF must extend to $m_{\rm max} \gg 10\,M_\odot$ and the total number of stars must be sufficiently large, $N \gg 10^3$. Otherwise there will simply not be enough (or any) massive stars to collide.

in the absence of significant mass loss from stellar evolution. Remarkably, we find that this mass is always (within the same "reasonable" assumptions about the IMF and initial cluster structure as above) around 10^{-3} of the total cluster mass, as suggested by observations (see Kormendy, Richstone, and van der Marel, this volume).

1.2 Monte Carlo Simulations of Dense Star Cluster Dynamics

Mass segregation and collisional runaways are driven by the most massive stars in a cluster, which, for a "normal" IMF, represent a very small fraction of the stellar population. For instance, in a Salpeter IMF from 0.2 to $120\,M_{\odot}$, only a fraction $\simeq 3\times 10^{-4}$ of stars are more massive than $60\,M_{\odot}$. Resolving these processes thus requires numerical simulations with very large numbers of particles (at least $\sim 10^5 - 10^6$). In the last few years, new MC codes have been developed that make possible simulations of stellar clusters with such high resolutions. The results reported here have been obtained with two independent MC codes that we call, for convenience, `MCglob` and `MCnucl`, as they were devised to follow the evolution of globular clusters and galactic nuclei, respectively. Both are based on the scheme first proposed by Hénon (1973), and they rely on very similar principles that we summarize below. In Sections 1.2.1 and 1.2.2, a few important aspects specific to each code are described.

The MC technique assumes that the cluster is spherically symmetric and can be modeled by a set of discrete particles. Each particle in the simulation could represent an individual star (as in `MCglob`), or an entire spherical shell of stars sharing the same orbital and stellar properties (as in `MCnucl` and most earlier MC codes). In the latter case, the number of particles may be lower than the number of stars in the simulated cluster, but the number of stars per particle has to be the same for all particles. Another important assumption in these codes is that the system is always in dynamical equilibrium, so that orbital time scales need not be resolved and the natural time step is a fraction of the relaxation (or collision) time. Instead of being determined by integration of its orbit, the position R of a particle (star, or the radius R of the shell) is picked at random, with a probability density that reflects the time spent at R: $\mathrm{d}P/\mathrm{d}R \propto 1/V_{\rm r}(R)$ where $V_{\rm r}$ is the radial velocity along the orbit.

The relaxation is treated as a diffusive process in the usual FP approximation (Chandrasekhar 1960; Binney & Tremaine 1987). The long-term effects on orbits of departures of the gravitational forces from a smooth quasi-stationary potential, $\phi_{\rm s}$, are assumed to be those of a large number of uncorrelated small-angle scatterings. If a particle of mass M_1 travels with relative velocity $\upsilon_{\rm rel}$ through a homogeneous field of particles of mass M_2 with number density n during δt, in the center-of-mass reference frame, its trajectory will be deflected by an angle $\theta_{\delta t}$ with

$$\langle\theta_{\delta t}\rangle = 0 \quad \text{and} \quad \langle\theta_{\delta t}^2\rangle = 8\pi \ln\Lambda\, G^2 n\,(M_1+M_2)^2\, \upsilon_{\rm rel}^{-3}\,\delta t,$$

where G is the gravitational constant and $\ln\Lambda \simeq 10-15$ is the familiar Coulomb logarithm (Binney & Tremaine 1987). In the MC codes, at each time step, a pair of neighboring particles are selected and their velocities are modified through an effective hyperbolic encounter with deflection angle $\theta_{\rm eff} = \sqrt{\langle\theta_{\delta t}^2\rangle}$. As any given particle will be selected many times, at various positions on its orbit, the MC scheme will actually integrate the effect of relaxation over the particle's orbit and over all possible field particles. Proper averaging is ensured if the time steps are sufficiently short for the orbit to be significantly modified only after a large number of effective encounters. The gravitational potential ϕ_s is approximated as the potential generated by all the particles. This potential is not completely smooth because the particles are razor-thin spherical shells whose radii change discontinuously. Through test computations, it can be shown that the corresponding spurious relaxation is negligible if the number of particles is $\gtrsim 10^4$ (Freitag & Benz 2001).

In contrast to methods based on the direct integration of the FP equation in phase space, the particle-based MC approach allows for the natural inclusion of many additional stellar and dynamical processes, such as stellar evolution, collisions and mergers, tidal disruptions, captures, large-angle scatterings, and strong interactions with binaries. The dynamical effects of binaries (the dominant 3- and 4-body processes), which may be crucial in the evolution of globular clusters, have been included in several MC codes through the use of approximate analytic cross sections and simple recipes (Stodołkiewicz 1986; Giersz & Spurzem 2000; Fregeau et al. 2003). Codes are currently in development by Giersz, Rasio and collaborators that will treat binaries with much higher realism by explicitly integrating 3- or 4-body interactions on the fly, a "brute-force" approach required to tackle the full diversity of unequal-mass binary interactions.

Among methods that can be used to follow the dynamical evolution of collisional stellar systems, only direct N-body integrations do not require assumptions on the geometry of the system or dynamical equilibrium and, being also particle based, rival the ability of MC codes to incorporate realistic physics. Unfortunately, as they are based on explicit integration of the orbits of N particles and require the computation of all 2-body forces, they are extremely computationally demanding, with a CPU time scaling like N^{2-3}. In practice, even with the use of special-purpose computers (such as the current GRAPE-6) and in spite of continuous progress in the development of N-body algorithms, the simulation of a cluster containing $\sim 10^5$ stars still requires months of computer time (Makino 2001). In contrast, MC codes, with CPU times scaling like $N\ln(N)$, routinely use up to a few million particles. Such high numbers of particles imply that globular clusters can actually be modeled on a star-by-star basis (Giersz 1998, 2001; Joshi et al. 2000, 2001; Watters et al. 2000). However, galactic nuclei typically contain $N_* \approx 10^7 - 10^8$ stars, and, for such systems, one has to take advantage of the fact that each physical process is included in the simulation with its explicit N_* scaling so that a single particle can represent many stars (in `MCnucl`).

1.2.1 A Monte Carlo Code for Globular Cluster Simulations

One of our MC codes, `MCglob`, based directly on the ideas of Hénon (1973) described in the previous section, was developed to study the dynamical evolution of globular clusters using a realistic number of stars (Joshi et al. 2000, 2001; Fregeau et al. 2003). One important characteristic of this code is that each particle represents a single star (or binary star) in the cluster, which allows us to incorporate stellar evolution processes in a completely realistic manner. The code uses a single time step for the evolution of all the stars (typically a small fraction of the core relaxation time), which allows for effective parallelization of the algorithm. Typical simulations for $\sim 10^5$ stars over $\sim 10^{10}$ yr can be performed in just a few hours of computing time.

This code has been tested extensively using comparisons to previous FP and direct N-body results, and it has been used to study a variety of fundamental dynamical processes such as the Spitzer instability (Watters et al. 2000) and mass segregation (Fregeau et al. 2002) in simple two-component clusters. It was also used to re-examine the question of globular cluster lifetimes in the tidal field of a galaxy (Joshi et al. 2001). Work is in progress to incorporate a full treatment of primordial binaries, including all dynamical interactions (binary–binary and binary–single) as well as binary stellar evolution (Rasio et al. 2001; Fregeau et al. 2003). Another major advantage of these star-by-star MC simulations (e.g., compared to direct FP schemes) is that it is straightforward to include a realistic, continuous stellar-mass spectrum. With a broad IMF, this requires adjusting carefully the Coulomb logarithm, which can be done by using the results of large N-body simulations for calibration (Giersz & Heggie 1996, 1997). Another difficulty introduced by a broad mass spectrum is the necessity of adjusting carefully the time step to treat correctly encounters between stars of very different masses. When pairs of stars are selected to undergo an effective hyperbolic encounter as described above, one has to make sure that the deflection angle remains small for *both* stars. In situations where the mass ratio of the pair can be extreme, one has to decrease the time step accordingly (Stodołkiewicz 1982). In practice, for the simulations described here, we find that the time step has to be reduced by a factor of up to $\sim 10^3$ compared to what would be appropriate for a cluster of equal-mass stars.

1.2.2 A Monte Carlo Code for Galactic Nuclei Simulations

To the best of our knowledge, `MCnucl` is the only Hénon-like MC code specifically aimed at the study of galactic nuclei (Freitag 2001; Freitag & Benz 2001, 2002b). In addition to relaxation and stellar evolution, it also incorporates a detailed treatment of collisions between single main-sequence stars. A central massive BH can also be included (together with a full treatment of tidal disruptions and stellar captures) but this is not necessary for the work presented here.

Individual time steps are used, with particles updated more frequently where the evolution is faster. Specifically, the time steps are set to some small fraction f of the *local* relaxation (or collision) time: $\delta t(R) \simeq f\left(t_{\rm rel}^{-1}+t_{\rm coll}^{-1}\right)^{-1}$. At each step, a pair of neighboring particles is selected randomly with probability $P_{\rm selec} \propto 1/\delta t(R)$, ensuring that the *average* residence time at R is $\delta t(R)$. After a particle is modified, the potential, stored in a binary-tree structure, is updated.

Unlike relaxation, collisions cannot be treated as a continuous process. They are discrete events that can affect very significantly the orbits and masses, or even the existence of particles. When a pair is selected, the collision probability between stars from each particle (shell),

$$P_{\rm coll} = S_{\rm coll} v_{\rm rel} n\,\delta t \quad \text{with} \quad S_{\rm coll} = \pi b_{\rm max}^2 = \pi (R_1+R_2)^2\left(1+\frac{2G(M_1+M_2)}{(R_1+R_2)v_{\rm rel}^2}\right),$$

is compared with a uniform-deviate random number to decide whether a collision has occurred. If so, the impact parameter b is determined by picking another random number X, with $b = b_{\rm max}\sqrt{X}$. The other parameters of the collision, i.e., M_1, M_2 and $v_{\rm rel}$, are known from the particle properties. The final outcome of the collision (new velocities and masses) is determined very accurately by interpolation from a table containing the results of more than 14,000 3-D SPH (smoothed particle hydrodynamics) calculations of encounters between two main-sequence stars (Freitag 2000; Freitag & Benz 2002a, 2004).

The structure and stellar evolution of collision products is an intricate problem that has only been studied in some detail for low-velocity collisions, relevant to globular clusters (Sills et al. 1997, 2001; Lombardi et al. 2002). The main issues are the importance of entropy stratification versus collisional mixing, and the rapid rotation of merger remnants. In view of these difficulties, we use one of two simple prescriptions to treat the stellar evolution of collision products. *(1) Maximal rejuvenation,* where the remnant is assumed to be completely mixed and is brought back on the zero-age main sequence. As hydrodynamic simulations show only very little mixing, this assumption leads obviously to an overestimate of the stellar lifetime of collision products. *(2) Minimal rejuvenation.* Here we assume that, during a merger, the helium cores of both parent stars merge together, while the hydrogen envelopes combine to form the new envelope; no hydrogen is brought to the core. An effective age on the main sequence is given by adopting a linear dependence of the helium core mass on age.

1.3 Core Collapse in Young Star Clusters

We have used our code `MCglob` to study the onset of core collapse in young star clusters with a wide variety of initial structures and stellar IMFs (Gürkan et al. 2004). The results of a typical simulation are illustrated in Figs. 1.1–1.3. This simulation was performed for a cluster containing 2.5×10^6 stars with a Salpeter IMF between $m_{\rm min} = 0.2 M_\odot$ and $m_{\rm max} = 120 M_\odot$. The rapid mass segregation of the most massive stars toward the cluster center is evident, as is the abrupt onset of core collapse once the central region becomes dominated by massive stars. In Fig. 1.2 we show the evolution of the mean radius of stars in various mass bins. We see that the mass segregation starts right at the beginning of the evolution and proceeds at a steady rate, even though the overall mass distribution (total density profile) in the cluster is hardly changing. Significant changes in the Lagrange radii become apparent only when the heaviest stars reach the center of the cluster. This is because these heaviest stars account for only a very small fraction of the total cluster mass. In all our simulations we find that the ratio of the core collapse time to initial half-mass relaxation time, $t_{\rm cc}/t_{\rm rh}(0)$, is always within the range 0.05–0.20 so long as the heaviest stars have masses $\gtrsim 20$ times the average stellar mass in the cluster. We used this result [$t_{\rm cc}/t_{\rm rh}(0) \simeq 0.1$] in Section 1.1 to derive our simple criterion for the onset of a runaway. In Fig. 1.3, we show the core collapse in more detail. We see that massive stars representing about 0.1% of the total cluster mass are driving the final core collapse. This ratio of the collapsing core mass to total cluster mass, $M_{\rm cc}/M_{\rm tot}$, also appears to be confined to a narrow range, of about 0.1%–0.3%, in all our calculations to date. These were performed for both Plummer models and King models with varying initial concentration, and for a variety of IMFs including simple power-law IMFs with exponents in the range 2–3 (the standard Salpeter value being 2.35). We have also checked the robustness of this result by varying the number of stars and the time step in our simulations. As pointed out in Section 1.1, the apparent agreement between this ratio and the ratio of BH mass to total cluster mass in many observed systems is very encouraging. Work is in progress to study more systematically the dependence on initial cluster structure, including the possibility of initial mass segregation.

1.4 The Runaway Collision Process

The formation of a massive central object by runaway collisions and mergers has been demonstrated in idealized FP models of proto-galactic nuclei by QS90. Unfortunately, because of the limitations of FP codes, these authors had to rely on a highly simplified treatment of collisions (see below). Furthermore, they assumed an initial single-mass stellar population, which significantly reduces the impact of mass segregation and stellar evolution.

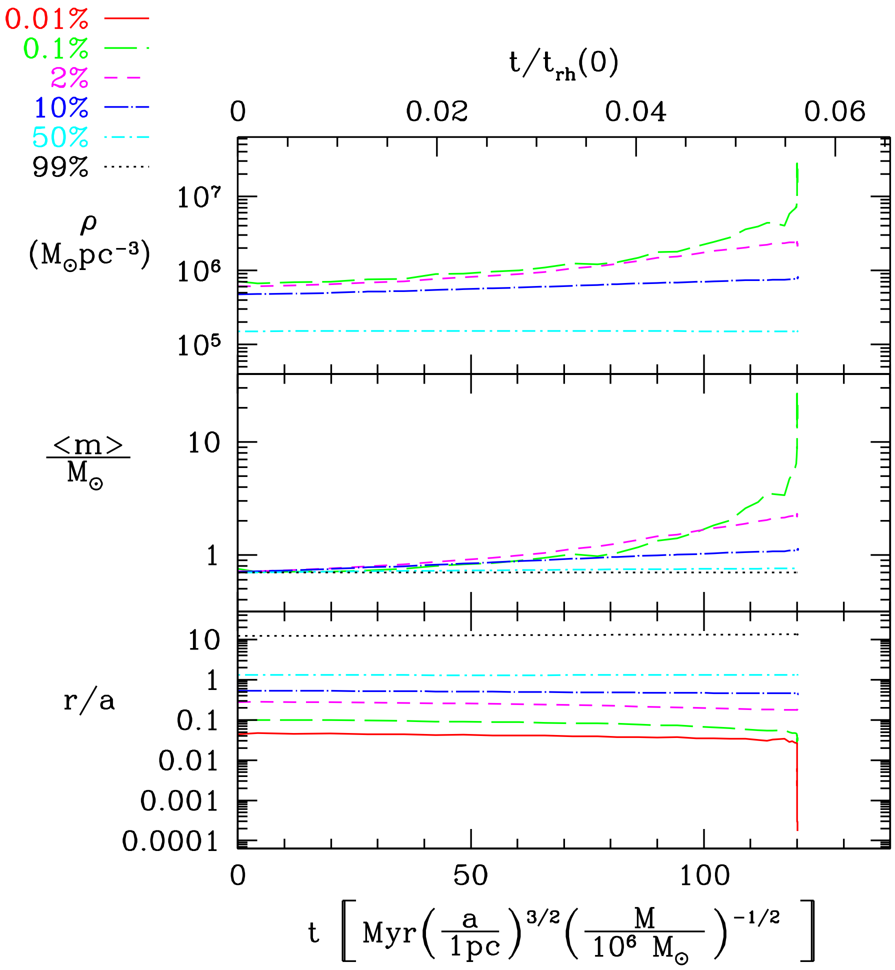

Fig. 1.1. The evolution of a cluster containing $N = 2.5 \times 10^6$ stars, terminated at core collapse. Time is given both in years (bottom) and in units of the initial half-mass relaxation time (top). The initial configuration is a Plummer sphere with a Salpeter IMF spanning the range from $m_{\rm min} = 0.2\,M_\odot$ to $m_{\rm max} = 120\,M_\odot$. The bottom panel shows the evolution of Lagrange radii (enclosing a constant fraction of the total cluster mass, indicated in the top left), in units of the Plummer length (core radius) of the initial model. The middle panel shows the average stellar mass within each Lagrange radius, and the top panel shows the average density within each Lagrange radius. Core collapse takes place at $t = 0.056\,t_{\rm rh}(0)$.

Indeed, these processes come into play only when more massive stars are formed through collisions.

Using `MCnucl`, we have started re-examining this problem with more realistic simulations. In the high-velocity environment of galactic nuclei, the formation and/or survival of binaries is unlikely (i.e., most binaries are "soft"), so it is reasonable to neglect them in the computations. As the stellar density rises abruptly to very high values during core collapse, collisions are bound to occur even in the absence of binaries.

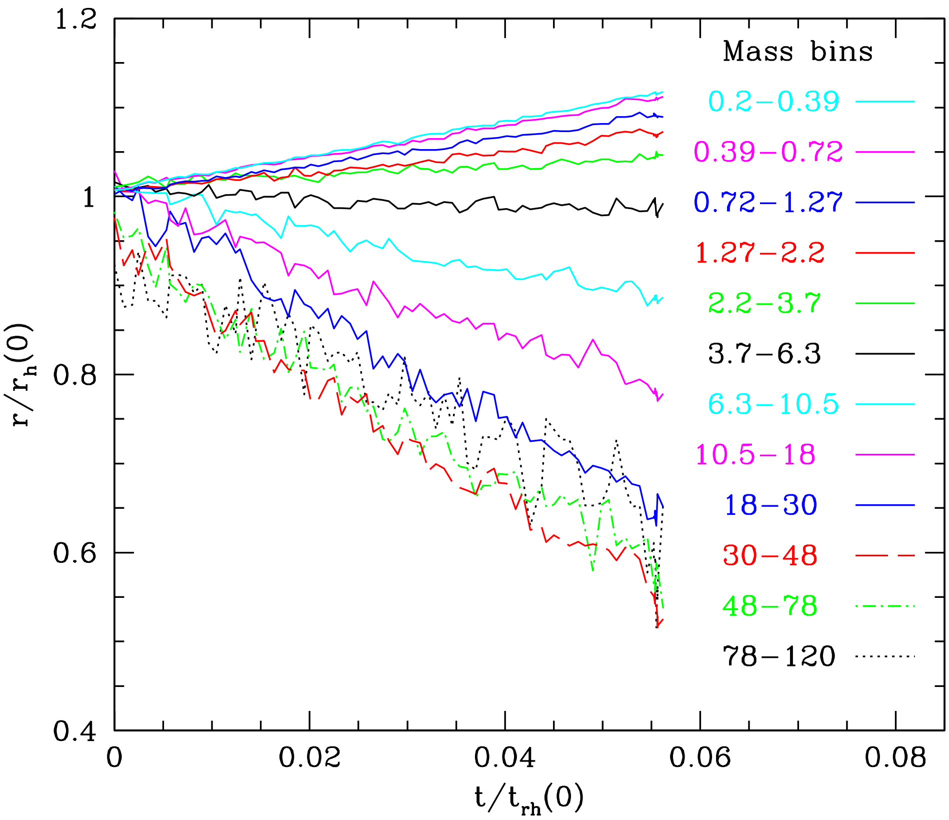

Fig. 1.2. The evolution of the mean radius (in units of the initial half-mass radius) for stars in various mass bins (indicated on the right; values are in units of $M_{\odot}$). The simulation is the same as in Fig. 1.1. Mass segregation is very clear, concentrating more massive stars near the center of the cluster.

For definiteness, we consider first QS90's model E4A, starting from a Plummer sphere with initial central density and 3D velocity dispersion of $3 \times 10^8\,M_{\odot}\mathrm{pc}^{-3}$ and $400\,\mathrm{km\,s}^{-1}$. QS90 started their FP simulations with all stars having $1\,M_{\odot}$ and assumed that all collisions lead to mergers with no mass loss and maximal rejuvenation. Not surprisingly, if we use the same, highly simplified treatment of collisions as QS90, we get clear runaway growth of one or a few particles. When we switch to our realistic prescription for the collisions and minimal rejuvenation, the runaway still occurs, although later. However, if we adopt a more realistic, Kroupa-type IMF (Kroupa 2001), significant mass loss from the massive stars occurs before core collapse has proceeded to high stellar densities. As we assume that the gas is not retained in the cluster, this mass loss drives a re-expansion of the whole system.

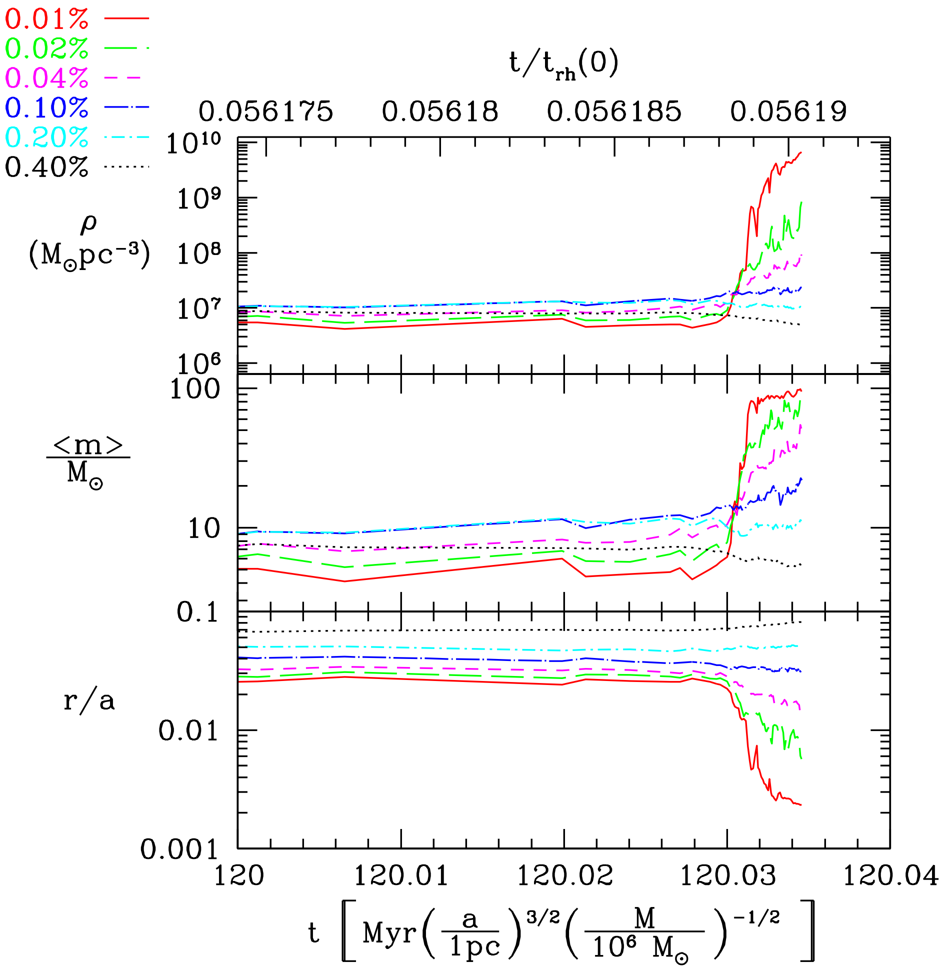

Fig. 1.3. Same as figure 1.1, but concentrating on the evolution of the cluster core near collapse.

A second, deeper core collapse occurs later, when the remnant (stellar-mass) BHs segregate to the center. The evolution of this dense cluster of stellar BHs cannot be treated with the present version of `MCnucl` because dynamically formed binaries will play a central role.

In addition to models with the same densities and velocity dispersions as the class “A” considered by QS90, we also simulated clusters with densities 3 times (models “Z”) and 9 times (models “Y”) larger (but the same velocity dispersion) with correspondingly shorter relaxation times (see Fig. 1.4). As shown in Fig. 1.5, models of class “Z” have a core collapse time slightly larger than 3 Myr and do not enter a runaway phase. One the other hand, runaway growth occurs in all simulations for clusters of class “Y,” which collapse in about 1 Myr. Figure 1.6 shows such a case. The growth of the runaway particle(s) is limited to a few hundred $M_\odot$ (650 $M_\odot$ in the “best” case), probably by some numerical artifact. Note that 500,000 particles were used for all these computations, independent of

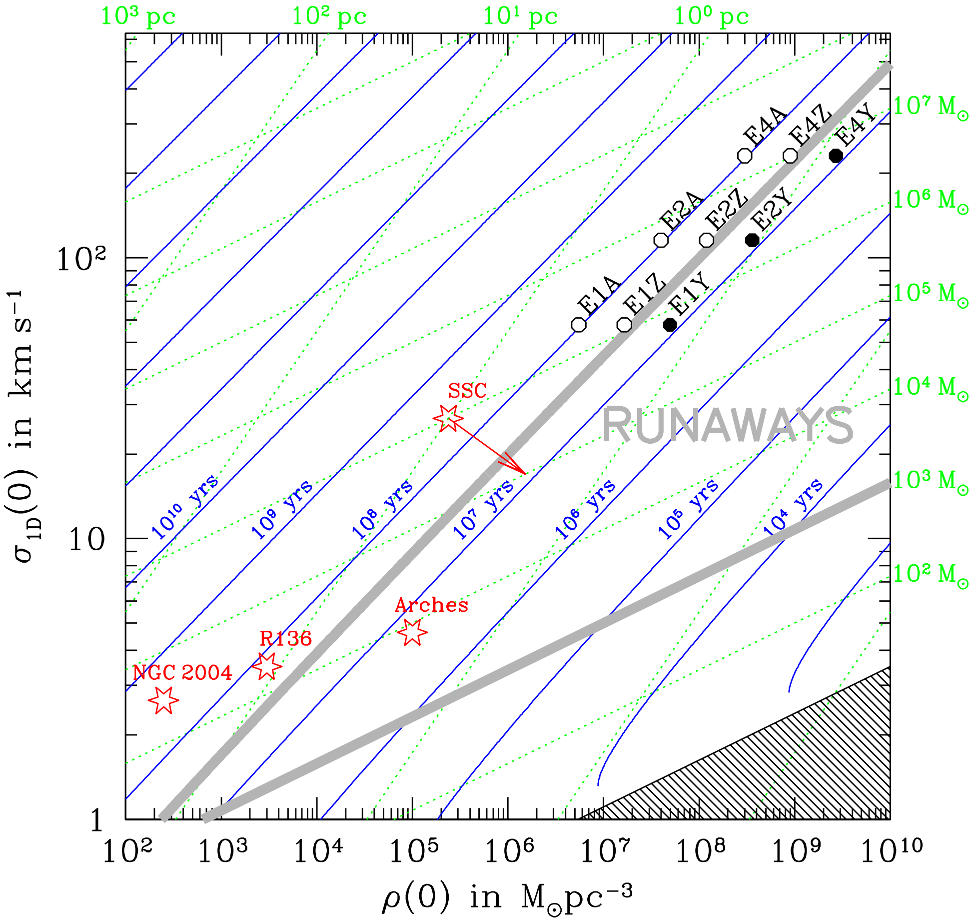

Fig. 1.4. Initial conditions for the clusters considered in Sec. 1.4. We use a notation similar to that of QS90, but our models have a broad IMF. Here $\rho(0)$ and $\sigma_{1D}(0)$ are the initial central density and velocity dispersion, respectively. The thin dotted lines show the values of the total mass (right labels) and the initial Plummer scale (top labels). We also show lines of constant half-mass relaxation time (solid lines labeled from 10^4 to 10^{10} yrs). Clusters born in the region between the thick diagonal lines have a core collapse time shorter than the lifetime of their most massive stars and are expected to undergo a runaway collision process. The solid and open round dots in the upper right region show the results of our MC simulations: an open dot indicates that a runaway was avoided, while a solid dot indicates that the onset of runaway collisions was detected. The open star symbols indicate the positions of a few observed young star clusters. The one labeled "SSC" indicates the position of a typical "super star cluster." Most of these systems have sizes at the resolution limit of the observations, with an upper limit on their radius of $\sim 1\,\mathrm{pc}$.

the true number of stars in the cluster. Hence, every particle represents many stars (12 to 36), a numerical treatment whose validity becomes obviously questionable as soon as a

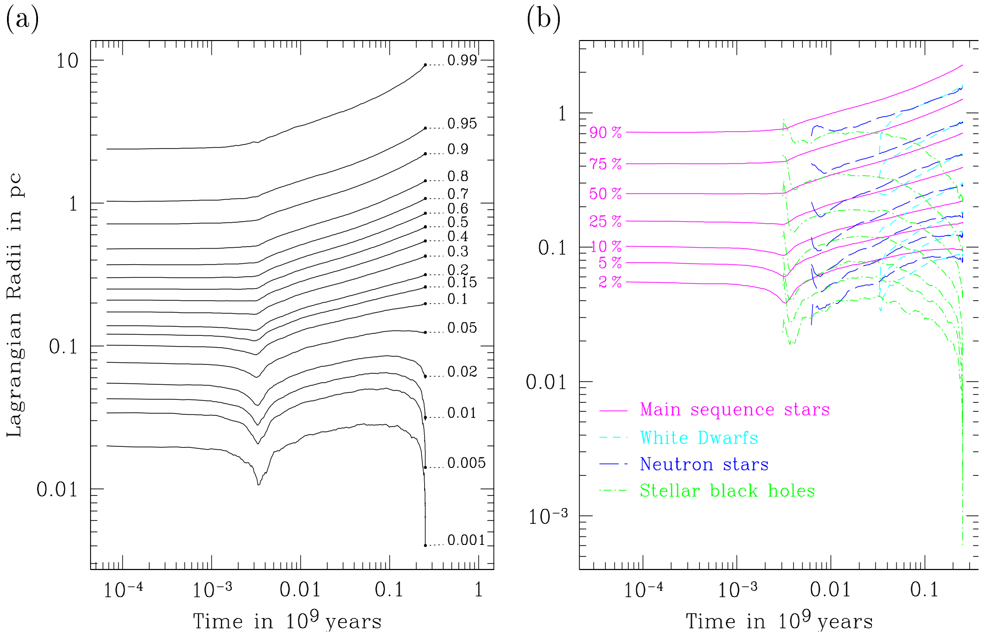

Fig. 1.5. Evolution of a cluster model with an extended IMF and an initial density 3 times higher than model E4A but the same velocity dispersion (see text). *(a)* Evolution of the overall Lagrange radii. The first, very mild core collapse is driven by the segregation of the massive stars to the center. It is quickly reversed by their evolution and the associated mass loss. The second, much deeper core collapse is driven by the stellar-mass BHs, which have become the most massive species after a few Myrs ($7\,M_\odot$). *(b)* Lagrange radii for the various stellar species.

single particle detaches from the overall mass spectrum. In addition, we note that, before the runaway abruptly saturates in our simulations, the growth rate observed is extremely rapid and the basic orbit-averaging assumption implied in the MC technique must probably break down. Possible physical processes that could terminate the runaway include: rapid mass loss from some of the massive stars, increased inefficiency of collisional merging for very massive objects*, depletion of the "loss-cone" orbits that bring stars to the center, or some combination of these factors. In any case, a robust conclusion can be drawn from these simulations, namely that runaway merging can produce stars at least as massive as $\sim 500\,M_\odot$ in the centers of clusters with $t_{\rm cc} \lesssim 3\,$Myr, even when central velocity dispersions are as high as $\sim 400\,{\rm km\,s^{-1}}$.

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* We have not computed collisions for stars more massive than $75\,M_\odot$, so considerable extrapolation of our results is required in these simulations.

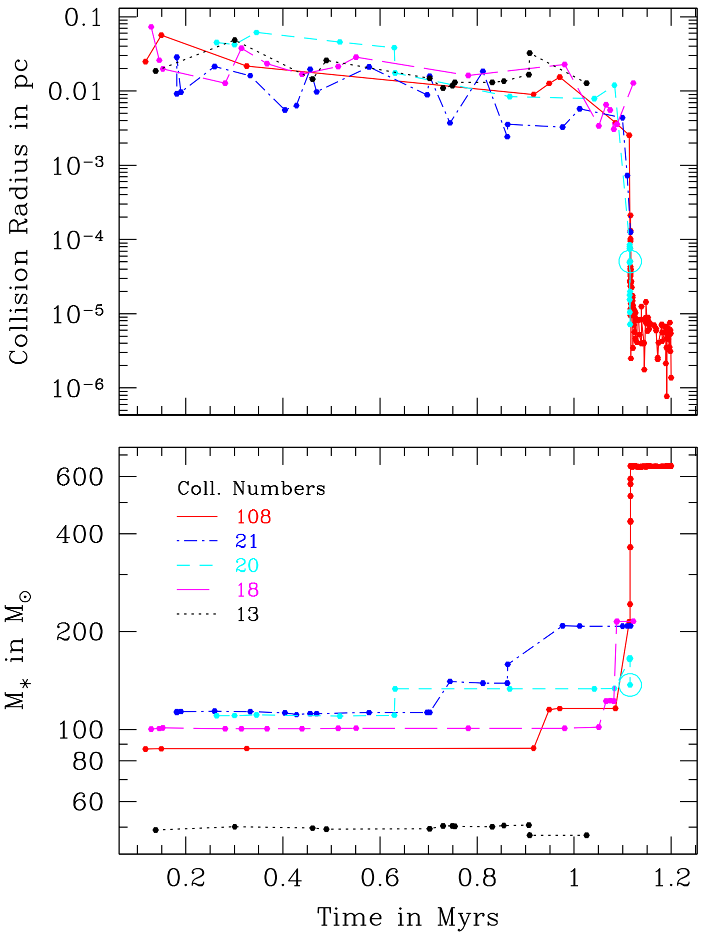

Fig. 1.6. Histories of a few particles that suffered from a large number of collisions. The initial model, E4Y, has an extended IMF and is 9 times denser than E4A but with the same velocity dispersion (see text). The top panel shows the distance from the center at which each collision occurred. In the bottom panel we plot the mass of the star after each collision. One particle (solid line) undergoes runaway growth up to about $650 M_\odot$. The reason why the growth saturates abruptly at this value is still unclear to us. The circle indicates that the particle has merged with a more massive one, probably the runaway star.

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