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Influence of Black Holes on Stellar Orbits

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Abstract
We review the current state of dynamical modeling for galaxies in terms of being able to measure both the central black hole mass and stellar orbital structure. Both of these must be known adequately to measure either property. The current set of dynamical models do provide accurate estimates of the black hole mass *and* the stellar orbital distribution. Generally, these models are able to measure the black hole mass to about 20%–30% accuracy given present observations, and the stellar orbital structure to about 20% accuracy in the radial to tangential dispersions. The stellar orbital structure of the stars near the galaxy center show strong tangential velocity anisotropy for most galaxies studied. Theoretical models that best match this trend are black hole binary/merger models. There is also a strong correlation between black hole mass and the contribution of radial motion at large radii. This correlation may be an important aspect of galaxy evolution.

1.1 Introduction

The first observational evidence that black holes are common in the centers of nearby galaxies is reviewed in Kormendy (1993) and Kormendy & Richstone (1995). The initial studies concentrate mainly on measuring the black hole mass and only somewhat included the effects of different orbital structure. However, it was always apparent that the assumed form for the distribution function has a considerable effect on the measured black hole mass. Thus, the believability in the existence of a central black hole closely paralleled the development of more sophisticated modeling techniques that were designed to be as general as possible.

There are two main aspects for making a general dynamical model. These are the dimensionality of the potential and that of the velocity ellipsoid. For the potential, we know that we have to at least model galaxies as axisymmetric, and, for some, triaxial structure is required (e.g., those with counterrotating cores, polar rings, etc.). While it is important to allow the most freedom for a dynamical model, there is a level of detail that need not be studied (at least at present). For example, we know that no galaxy is exactly symmetric along any axis. Therefore, in order to provide an adequate representation in that case, one cannot use symmetric dynamical models, but instead must rely on N-body simulations—similar to what is done when modeling merging systems (Barnes & Hernquist 1992). Using an N-body system to model each galaxy is currently not practical, and, furthermore, may not even provide a better understanding of the underlying physics due to the huge parameter space inherent

Dimension of Velocity Ellipsoid

parametric
nonparametric

Dimension of Potential

	1D Isotropic	2D r, θ	3D r, θ, ϕ
1D Spherical	King models Plummer many, many others *Gebhardt & Fischer (95)*	King (66), Michie (63) Merritt (85), Osipkov (79) *Binney & Mamon (82)* *Merritt & Gebhardt (94)* *Rix et al. (97)*	*Richstone & Tremaine (84)*
2D Axisymmetric	Isotropic rotators	Toomre (82), scale−free Richstone (84), scale−free Binney et al. (90) van der Marel (91) Magorrian et al. (98) *Merritt et al. (97), edge−on*	Kuzmin & Kutuzov (62) Dejonghe & de Zeeuw (88) Levison & Richstone (87) *Gebhardt, Richstone (03)* *van der Marel et al. (98)* *Cretton et al. (99)* *Verolme et al. (02)*
3D Triaxial		Stackel Potential de Zeeuw (85), Statler (87) *Zhao (96)* *Verolme et al. (03)*	

Fig. 1.1. The two main assumptions made in dynamical models: the dimension of the potential is along the vertical axis and that of the velocity ellipsoid is along the horizontal axis. Each box includes a few relevant papers for each configuration. These references are not complete and only serve to provide examples. The text type refers to whether the dynamical models assume a parametric (regular) or nonparametric (italics) form for the distribution function.

in *N*-body simulations. Thus, the most to gain lies in using general dynamical models that may not accurately represent the galaxy, but serve to provide overall trends from which we can infer formational and evolutionary scenarios. In other words, we will always be making some error—no matter which dynamical models we use—but we should be aware of each model's limitations. Below, we first review the dynamical models that have been applied to nearby galaxies, and then summarize the current state-of-the-art and the overall results from these models.

1.2 The Suite of Dynamical Models

Figure 1.1 diagrams the possible dynamical models based on their complexity. The components are the number of symmetry axes for the potential and for the velocity ellipsoid. The potential shapes clearly represent spherical, axisymmetric, and triaxial shapes.

The velocity ellipsoid shapes represent isotropic (distribution functions that depend on only one integral of the motion, namely energy), 2-integral, and 3-integral distribution functions. This plot provides the range of possible distribution functions that can be used for dynamical modeling where the system obeys some symmetry axes. Obvious omissions are those systems that obey no symmetry axes.

The goal of the dynamical modeling is to determine the underlying potential of the system as well as the orbital structure. The concern is that, by not using a model that adequately represents the system, the results may be significantly biased. The best way to test for these biases is to model systems with a variety of assumptions and compare the results.

In each grid element are examples from the literature that represent that particular model. This listing is done to provide a few examples each and is not intended to be complete in any way. In fact, a complete listing would take the whole of this proceeding (but see Binney & Tremaine 1987 for a complete discussion). There are two types for the text in each grid: regular text represent analytic models, and italicized text represents nonparametric models. For example, isotropic, spherical models (the upper left grid) encompass an infinite number of density-potential pairs, and only King and Plummer models are listed. Gebhardt & Fischer (1995) present a nonparametric, isotropic, spherical model that determines the potential directly. Isotropic, spherical models have been enormously successful in describing stellar systems, especially for globular clusters (King 1966). For measuring black hole masses, they have done remarkably well; for example, Kormendy (2004) shows the change in the estimated black hole mass for M32 varies little over 20 years of data and a range of model sophistication. However, we are at a level now where the quality of the data is so high that we must use the most general models possible. Furthermore, in order to study the orbital structure one must use nonparametric techniques; otherwise, one restricts the form of the distribution function.

Spherical models are good representations for globular clusters and some of the largest ellipticals (e.g., M87), but we know that most galaxies are not spherical. Tremblay & Merritt (1995) and Khairul Alam & Ryden (2002) argue, based on inversion of the distribution of projected shapes, that, in fact, there are nearly no galaxies that are spherical. We must use, at the least, axisymmetric models. Furthermore, Binney (1978) and Davies et al. (1983) point out that the flattening in galaxies is not consistent with isotropic orbits: i.e., we must also include anisotropy. Thus, there have been a tremendous amount of work in modeling galaxies as 2-integral axisymmetric systems.

Van der Marel (1991) provides one of the first 2-integral studies of a large sample of ellipticals, using the modeling first introduced by Binney, Davies, & Illingworth (1990). From kinematic data taken along the major and minor axis for 37 galaxies, van der Marel finds that 2-integral models have too much motion on the major axis compared to what is seen. The implication is that ellipticals have $\sigma_r > \sigma_\theta$, inconsistent with the 2-integral assumption (where $\sigma_r = \sigma_\theta$). There are multiple ways to cause this inconsistency. For example, galaxies may depend on a third integral of motion; the 2-integral models may be biased by not including a dark halo; galaxies may have significant triaxial shape which also biases axisymmetric models; or the quality of the data may be too poor. We can compare the results of van der Marel to those of Gebhardt et al. (2003), who make 3-integral models for 12 galaxies, including three in common. For half of the sample, $\sigma_r > \sigma_\theta$, consistent with van der Marel. For three galaxies in common (NGC 3379, NGC 4649, and NGC 4697), there is not good agreement. Neither model includes a contribution from a dark halo, which

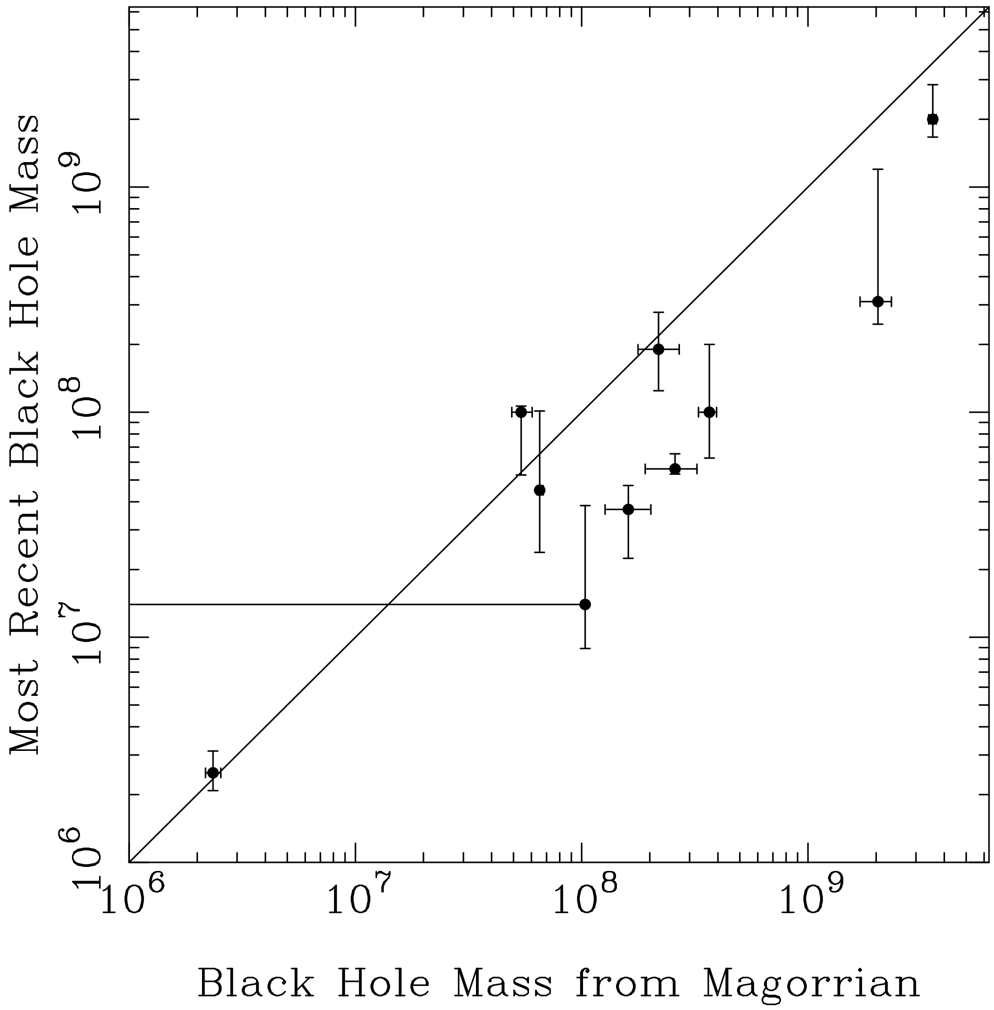

Fig. 1.2. The black hole mass estimate from Magorrian et al. (1998) and those from more recent data and analysis (see Tremaine et al. 2002 for the compilation). The line is one-to-one correspondence. All masses have been corrected to a common distance. The Magorrian et al. (1998) masses are on average 2.4 times larger.

may bias the large radial orbital structure. However, most likely, the differences are due to the use of different data sets, and the quality of the data sets can have a significant influence on the results. We now turn to measuring the black hole mass.

The largest sample using 2-integral models to measure black hole masses is that of Magorrian et al. (1998). Magorrian et al. study 36 galaxies with ground-based kinematics and *HST* photometry to provide a systematic estimate of the central black hole mass. Previous black hole studies concentrated on individual cases. These result have been widely used, and also criticized. The major complaint is that the models are still too simplistic (i.e., 2-integral) and that the kinematic data have too low spatial resolution to say anything about the central black hole. Many of the Magorrian et al. galaxies now have *HST* kinematic data and have been modeled with more general models. In Figure 1.2 we compare the black hole mass estimates from Magorrian et al. to these more recent studies. There is a bias in that the 2-integral masses tend to be higher than those from the more recent analysis. The average difference between the two samples is a factor of 2.4. As discussed in Gebhardt et al. (2003), the difference appears to be due to differences in modeling, as opposed to the improved spatial resolution in the kinematics. Clearly, the better kinematics provide a more accurate measurement, but they do not appear to bias the results. Gebhardt et al. (2003) show that the black hole mass is not biased when using only ground-based data compared to using both ground-based and *HST* kinematics.

In order to provide a more accurate estimate of either the black hole mass or the orbital structure, we need to go beyond 2-integral models. Models that allow for three integrals of motion have only recently been applied to dynamical systems. The problem is that the most general form for the third integral is not analytic, and we must rely on numerical approaches. In limiting cases, there are analytic 3-integral models; for example, Dejonghe & de Zeeuw (1988) study 3-integral Kuzmin-Kutuzov (1962) models. However, these models have analytic cores ($d\log\nu/d\log r = 0$ at the center), and since nearly all galaxies have central cusps (Gebhardt et al. 1996; Ravindranath et al. 2001), they will be of limited use. Because the third integral is not analytic, we generally rely on orbit-based, Schwarzschild (1979) codes in order to study them. The first general application of the orbit-based methods is presented in Richstone & Tremaine (1984), applied to spherical systems. They even incorporate rotation in their models to provide one of the first models that include three integrals (energy, E, total angular momentum, L^2, and angular momentum about the pole, L_z), albeit in a spherical system. Rix et al. (1997) extend this analysis to make a detailed orbit-based model of the dark halo around NGC 2434. The first application of an axisymmetric, orbit-based model is that of van der Marel et al. (1998), who measure the black hole mass in M32. A few groups now have axisymmetric, orbit-based codes that have been used to study central black holes. To date, 17 galaxies have been studied with these models, with 14 coming from one code (Gebhardt et al. 2000, 2003; Bower et al. 2001), four from the Leiden group with various codes (van der Marel et al. 1998; Cretton & van den Bosch 1999; Cappellari et al. 2002; Verolme et al. 2002), and one from Emsellem, Dejonghe, & Bacon (1999).

With so few groups using orbit-based codes, we must be certain that the immense freedom allowed by these codes does not bias the results due to some feature of an individual code. The general problem of covering phase space appropriately in these orbit-based codes is tricky. There is a balance that one must obtain between including a large orbit library in order to sample phase space but still maintain a small enough library in order to use a reasonable amount of computer resources. In fact, Valluri, Merritt, & Emsellem (2004) find that there is a large difference when running models using orbit libraries of various size. They have two main results that question the reliability of these models for measuring black hole masses. First, the shape of the χ^2 contours depends on the number of orbits run for a model, using the same data set. Second, for models with large numbers of orbits, there is a degeneracy in black hole mass: the χ^2 contours reach a plateau over a large range of black hole masses. These results are critically important to understand since they may undermine this whole area of study. Fortunately, the other groups involved have done many tests in regards to this degeneracy. We will concentrate on the tests done with the Gebhardt et al. (2003) code.

There are three issues on which we will focus. These are (1) the shape of χ^2 as a func-

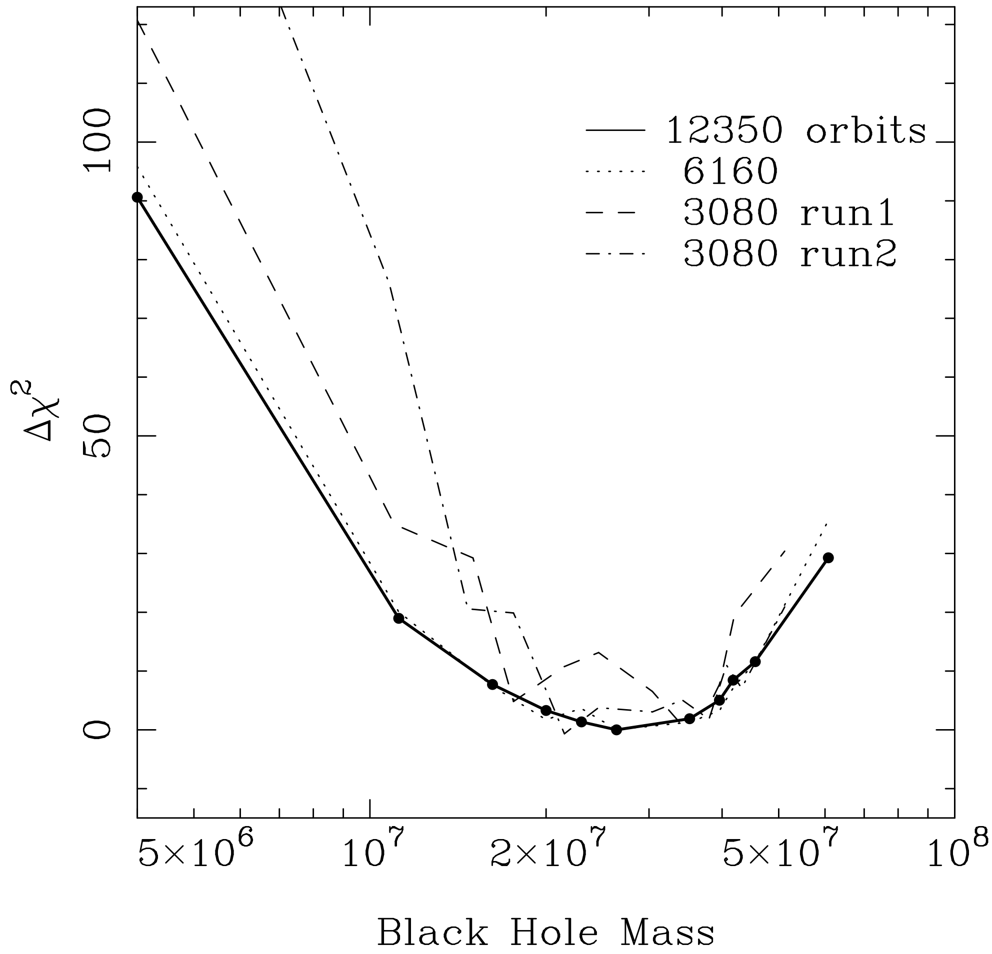

Fig. 1.3. Shape of $\Delta\chi^2$ versus black hole mass for models with different orbit numbers. Each model is a fit to an identical data set, and the only difference is the sampling of phase space. For the two runs with the smallest orbit library, we have run the same number of orbits, but simply sampled phase space differently.

tion of orbit number, (2) the ability of using the χ^2 contours to measure reliable confidence bands, and (3) the dependence on the smoothing parameter. For this last aspect, most groups use regularization for the smoothing while Gebhardt et al. rely on maximizing entropy (Richstone & Tremaine 1988). Regularization imposes smoothing directly in phase space by including a term that represents the noise in the χ^2, typically using the sum of the squared second derivative between phase space elements. Gebhardt et al. (2003) calculate the entropy of each orbit (using entropy equal to $w\log w$, where w is the orbital weight) and use the total entropy as a constraint (see Richstone & Tremaine 1984, 1988 for a complete discussion). Both approaches should provide smooth distribution functions, and there is no obvious desire to use one over the other.

To study the influence of orbit number on the best-fit solution, the obvious test is to run

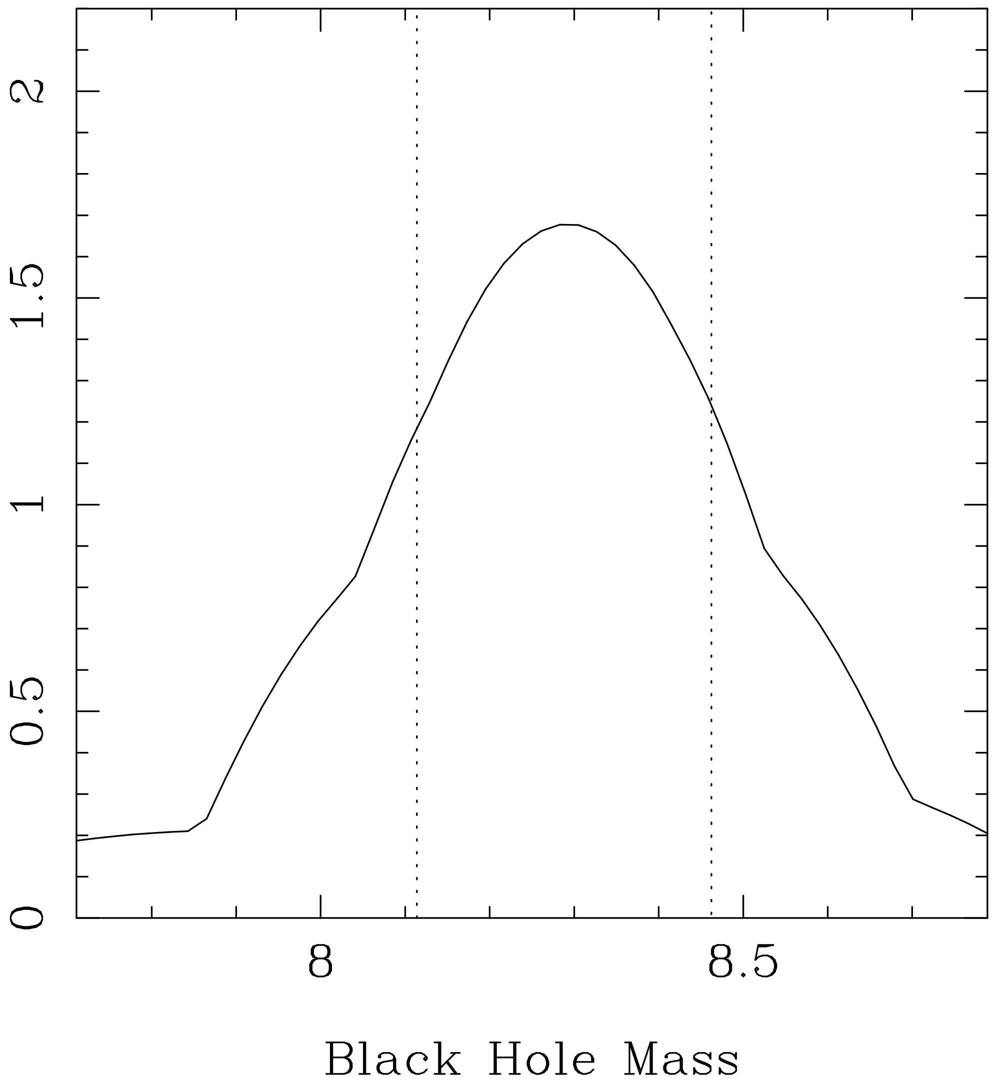

Fig. 1.4. Distribution of black hole masses from Monte Carlo simulations for NGC 3608. The solid line represents the results of changing the input velocity profiles according to the noise in the spectral data (the Monte Carlo approach). The vertical dotted lines represent the 68% confidence limit as measured from the shape of the χ^2 contours. The area inside the dotted lines is close to 68% of the area.

an analytic model where the black hole mass is known and simply increase the orbit number. Valluri et al. (2004) have the only paper in which this test has been published. This test, however, has been done by the other groups, but it was never published since nothing was ever seen to be problematic. Figure 1.3 plots this test using the code of Gebhardt et al. (2003). They run four models for the same data set. The total orbit number spans a factor of 4, with the two smallest libraries being run twice but with a different sampling. The two largest libraries show nearly identical χ^2 profiles. The two smallest libraries show a different contour shape, but they have substantial noise, making the comparison difficult. For libraries with an extremely small number of orbits, it is clear that the χ^2 contours must become very noisy since the quality of the fit depends on whether one happens to hit important orbits or not. Thus, having an appropriate number of orbits certainly is important. However, since we see little difference between the two largest libraries that differ by a factor of 2 in orbit number, it appears that the contours do not plateau as a function of black hole mass, as Valluri et al. (2004) find. In fact, even for the small libraries, we see that they tend to trace the true χ^2 contour fairly well, although the noise makes it difficult to follow. The number of orbits in a given library is only useful if one compares it to the number of model grid elements. For published orbit-based models, most have phase space coverage that is adequate to measure the black hole mass. For example, Gebhardt et al. (2003) use about 8000 orbits in each galaxy model with the same number of grid elements shown in Figure. 1.3.

Another issue to understand is whether the uncertainties on the black hole masses are adequately measured. One of the goals of black hole studies is to understand their role in galaxy evolution, and any comparison with galaxy properties must contain accurate uncertainties. All orbit-based models rely on using the shape of the χ^2 to determine their uncertainties. The best method, however, is to run bootstrap simulations on the real data. We have done this for NGC 3608. For each spectrum, we simulate a new realization based on the noise in the spectrum. We then generate 100 realizations. This Monte Carlo method is the same as that used when measuring the uncertainties for the velocity profiles (see Pinkney et al. 2004). We then run the modeling code on each new set of data and estimate the best-fit black hole mass. This procedure is extremely time consuming, and we have only done it for one galaxy so far. Figure. 1.4 plots the distribution of black hole masses obtained by these Monte Carlo simulations. The solid line represents the distribution function using an adaptive kernel estimate of the individual realizations. The dotted lines show the 68% confidence band measured from the shape of the χ^2 contours. The agreement is excellent, as the 68% χ^2 contours are similar to the area that contains 68% of the simulations. The simulations encompass a slightly larger area, but only by a few percent. Thus, it appears that the χ^2 contours can be used to estimate accurately the black hole mass uncertainties. From all of the orbit-based models used to date, the range of black hole mass uncertainties is from 5% to 70%, with an average uncertainty around 20%. Given that the scatter in the $M_\bullet - \sigma$ correlation is less than 30% (Tremaine et al. 2002), we still need to improve the black hole mass uncertainties.

Another important concern is whether the smoothing parameter has an effect on the black hole mass. The choice of this parameter is discussed extensively in Cretton et al. (1999) and Verolme et al. (2002). Their choice of the smoothing parameter is based on comparison with analytic test cases, by finding that smoothing parameter that provides the best match for the phase space distribution function. This cross-validation technique is a standard statistical approach to determine the smoothing parameter. Furthermore, Verolme et al. (2002) have performed tests in which they compare their best-fit mass found with optimal smoothing to that measured when including no smoothing, and find no difference in their black hole mass. Similar results are found in the modeling of Gebhardt et al. (2003). Figure. 1.5 is a plot of χ^2 versus smoothing parameter for many different models of NGC 3608. Each line differs by the mass of the black hole. The final χ^2 versus black hole mass is then obtained by taking the rightmost values in Figure. 1.5. The point of this plot is to show that the best-fit model

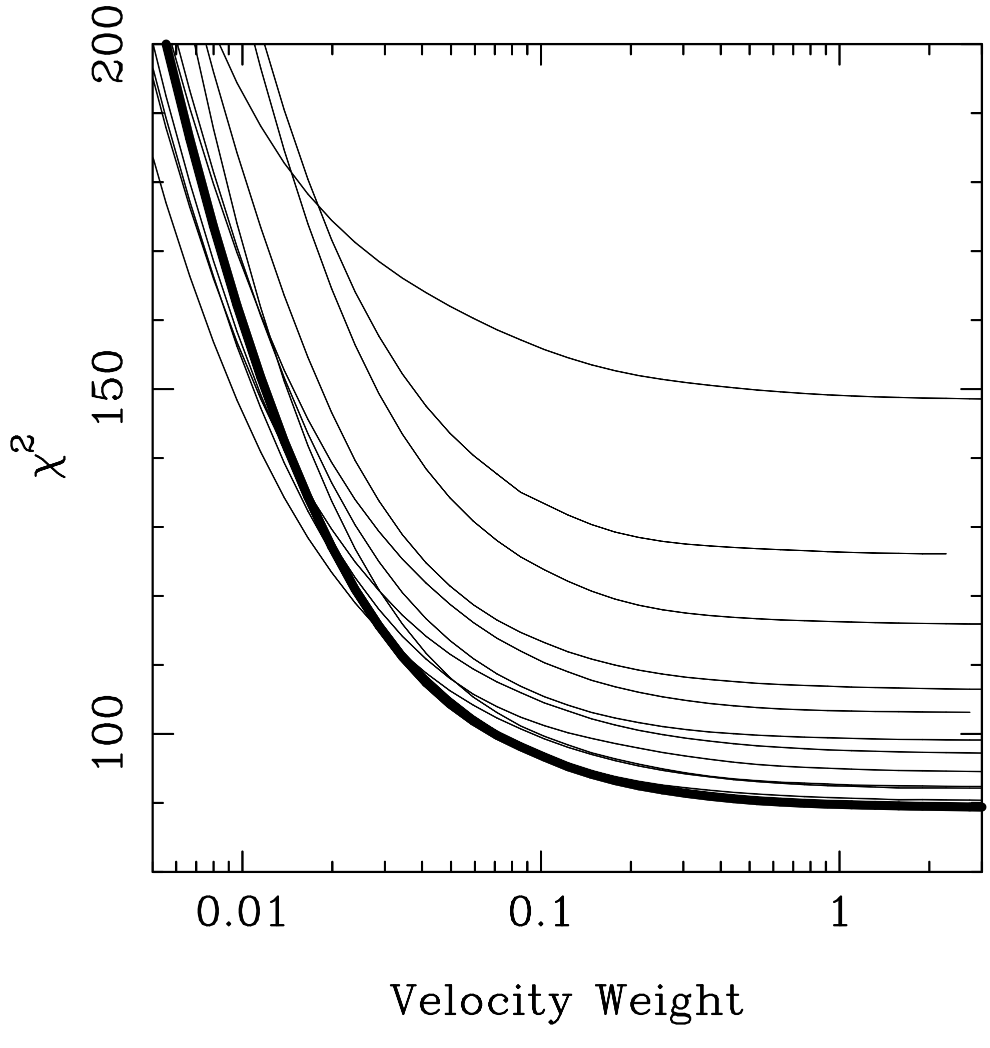

Fig. 1.5. χ^2 between the model and data as a function of the relative weight between entropy and velocity fit for NGC 3608. By increasing the velocity weight, we are decreasing the amount of smoothing. We start each model with maximum smoothing (maximizing entropy only) and then increase the velocity weight until the kinematics are fit as well as possible. The heavy, solid line is the best-fit model when the entropy term has no weight. The best-fit model has the minimum χ^2 over a large range of entropy weights.

provides the minimum χ^2 over a large range of smoothing parameters. For the maximum entropy method, the smoothness is employed by increasing the contribution of the entropy term relative to the comparison with the velocities. The velocity weighting is increased until the model provides the best fit to the data and essentially there is no contribution from the entropy term. However, as is seen in Figure. 1.5, the best-fit model provides the minimum χ^2 for a range of 100 in smoothing parameters.

All of the above discussion has focused on measuring the black hole mass and not the stellar orbital structure. The influence of these effects on the orbits is harder to quantify, since the results depend on which aspect of the orbits that concern us. For example, the answer depends on whether one is concerned with the velocity ellipsoid at every position in the galaxy, or whether one wants the radial to tangential components at only two different radii. The former is much harder to measure. We are not at the point where we can study the detailed shape of the velocity ellipsoid throughout the galaxy. The two ingredients required to do this are (1) an understanding of any systematic biases in the orbit-based techniques and (2) having the appropriate data sets to perform this analysis. We will discuss each of these below, but at this point we stress that obtaining a simple measure of radial to tangential motion appears to be robust, and does provide evolutionary constraints. Using the same Monte Carlo simulations discussed above, we can also estimate the distribution of radial to tangential motion from the noise in the spectra. The scatter is remarkably small. Similarly, this ratio has very little dependence on the smoothing parameter. In fact, that ratio changes by a much smaller fraction than the best-fit black hole mass. This quantity is typically measured to around 20% or better. Thus, we are confident that we can use this number to provide good comparison with theoretical predictions.

1.3 Results and Discussion

There are 17 galaxies that have axisymmetric orbit-based models. Figure 1.6 plots the orbital properties of those galaxies against other galaxy properties. We include the black hole mass, the effective dispersion, and the radial to tangential motion at two points in the galaxy—the central region and at 1/4 effective radius. In the central region for each galaxy, the black hole dominates the potential. The $M_\bullet - \sigma$ plot is the most significant correlation. However, there is also a very strong correlation between the black hole mass and the radial motion contribution at large radii (top right plot). There is another correlation of this quantity with effective σ , but this may be secondary to the one with black hole mass. In fact, the correlation with black hole mass is the most significant of all other galaxy properties (total light, total mass, effective radius, etc.). The trend is that those galaxies that have large black hole masses (and hence large σ) have orbits dominated by radial motion at large radii. Tangential motion tends to occur in those galaxies with small black holes. This correlation is one of the strongest for the full set of comparisons in Gebhardt et al. (2003).

The correlation between $M_\bullet$ and σ_r/σ_{te} is likely to be related to the evolutionary history of the galaxy. For the most massive galaxies, at radii near to the effective radius, the orbital distribution is radially biased. This is also the conclusion from Cretton, Rix, & de Zeeuw (2000), who use orbit-based methods to study the giant elliptical NGC 2320; along the major axis, they find strong radial bias in the orbits at large radii. We can compare this radial bias for the most massive galaxies with the *N*-body simulations of Dubinski (1998). He finds that for the most massive ellipticals, there is an increase in the radial motion from the center (where it is nearly isotropic) to the outer radii (where the merger remnant has $\sigma_r/\sigma_\theta = 1.3$). The most massive galaxies in our sample of 17 approach this amount of radial motion at large radii. For the smaller galaxies, the *N*-body comparisons are not as developed for measuring the internal orbital structure. However, based on the recent results of *N*-body simulations (Meza et al. 2003; Samland & Gerhard 2003), we will soon be in a position to compare the internal structure of the smaller galaxies as well. It has long been known that low-luminosity ellipticals rotate rapidly and are often consistent with oblate isotropic rotators, while high-luminosity ellipticals have been thought to be supported by radial anisotropy at large radii

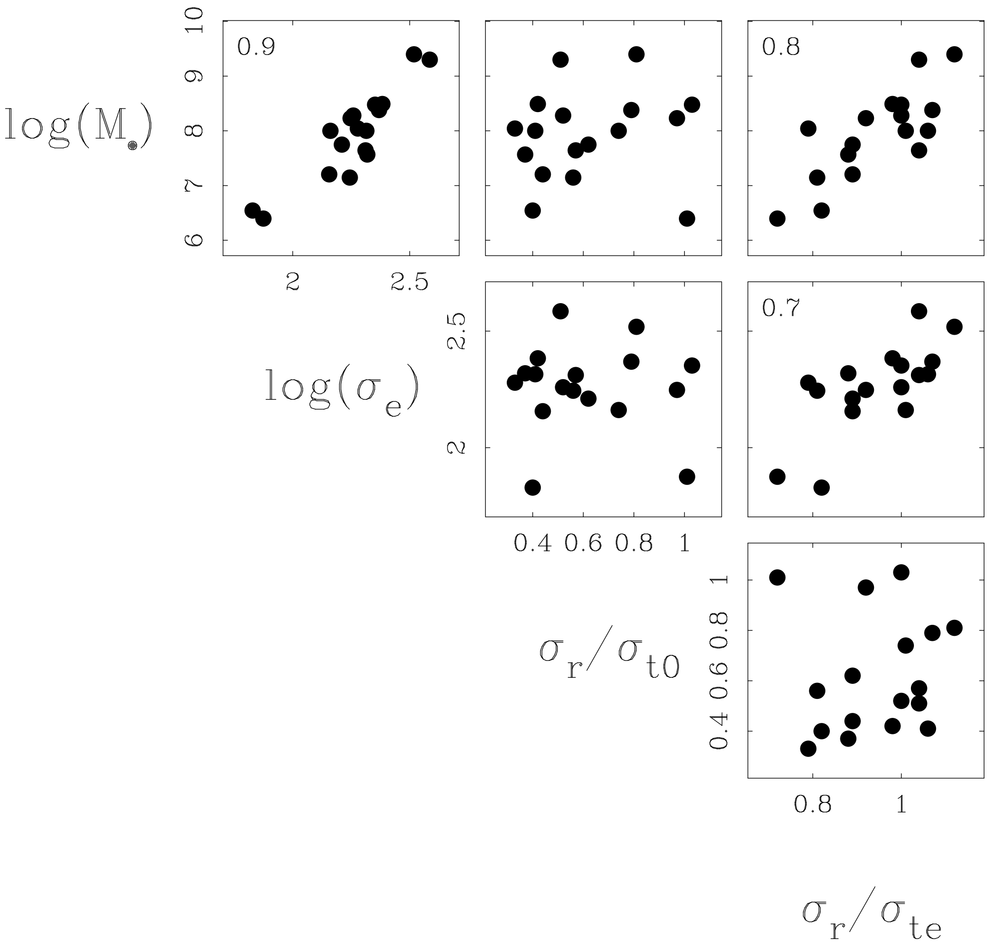

Fig. 1.6. Plots of the orbital properties against various galaxy properties. The properties along the diagonal include the black hole mass, the effective dispersion (σ_e), the ratio of radial to tangential dispersion at the center (σ_r/σ_{t0}), and the ratio of radial to tangential dispersion at 1/4 the effective radius (σ_r/σ_{te}). The number written in the upper left corner of the plot is the Pearson's *R* correlation coefficient. If the probability from the correlation is below 10%, we do not report *R*.

(Davies et al. 1983). Since black hole mass correlates with luminosity, the $M_\bullet - \sigma_r/\sigma_{te}$ correlation may then be secondary; however, the radial anisotropy correlates much stronger with black hole mass than it does with luminosity. There has been a considerable amount of theoretical work in explaining why the black hole mass correlates so well with host galaxy dispersion (see Adams et al. 2003 and references therein for a recent discussion). The correlation may provide additional constraints on the models.

There is also a trend that the galaxies with shallow central density profiles (i.e., the core galaxies) have orbits with the strongest tangential bias near their centers. This correlation has been discussed in Gebhardt et al. (2003). The most likely explanation is that this is caused by binary black hole mergers. We know that the existence of a black hole will leave some amount of tangential anisotropy since it will either eject or accrete those stars that are on radial orbits. This effect has been seen in many *N*-body simulations that consider adiabatic growth of black holes (Quinlan et al. 1995, 1997; Nakano & Makino 1999; Milosavljević & Merritt 2001; Sigurdsson 2004). In all of these case, however, the amount of tangential motion is quite small. In the most detailed study to date, Milosavljević & Merritt (2001) find that the most extreme amount of tangential motion has $\sigma_r/\sigma_\theta = 0.8$. The values that Gebhardt et al. (2003) report are smaller than 0.4. One way to obtain such large amounts of tangential motion is to have a binary black hole that can affect more stars on radial orbits due to its own orbital motion. The binary black hole results from a merger, and we already have seen that binary black holes are one of the best mechanisms to create the division between core and power-law galaxies (Faber et al. 1997; Milosavljević & Merritt 2001; Lauer 2004). However, the *N*-body simulations that have been studied use fairly restrictive assumptions—most are based on spherical isotropic initial conditions. Once realistic simulations including mergers and central black holes are available, we will be in a much better position to interpret the observational results.

1.4 The Future

There are many aspects of understanding the stellar orbital structure that need improvement—these include the data, analysis, and theoretical comparisons. In regards to the data, with the use of orbit-based models, we can realistically constrain the internal structure of the galaxy. In fact, Verolme et al. (2002) were able to measure with high accuracy the inclination of M32, and thus its intrinsic shape. However, in order to do this they needed two-dimensional kinematic data, which were obtained by the SAURON team (de Zeeuw et al. 2002). Most of the galaxies studied to date with orbit-based models only have limited kinematic data (along 2–4 position angles) and thus cannot be used to study their intrinsic shapes. In fact, as a result of this, most of the models in Gebhardt et al. (2003) are only run as edge-on configurations, and there is a concern that this may bias the results (de Zeeuw 2004). However, for the issues discussed here—the black hole mass and radial to tangential motion—inclined models are unlikely to introduce substantial changes, given the large uncertainties already on these quantities. In any event, significant improvement can be made by using two-dimensional kinematic data. Another area for improvement of the data is to include kinematics at large radii. In the study of Gebhardt et al., they were careful to report only results inside of the effective radii, where the dark halo is unlikely to have any influence. However, any dynamical model needs to include some estimate of the influence of orbits at large radii. Even though the effect of these orbits is expected to be minimal at small radii, they are not ignorable. In order to measure the central black hole and orbital structure, a proper dynamical model should include both high-spatial resolution (i.e., *HST*) and large-radii kinematics. With the advent of integral-field units on many large ground-based telescopes, obtaining this type of data will be feasible. In fact, adaptive optics observations with an integral-field unit will be a tremendous advance to this field of study.

On the data analysis side, while the orbit-based models that have been run offer significant improvement over the previous set of models, there is still a long way to go. For instance, most orbit-based models are axisymmetric and oblate. Prolate and triaxial models need to be included for a proper analysis. As discussed above, even for the oblate models, most include only an edge-on configuration. In addition, many have assumed luminosity density profiles that have constant ellipticity with radius. We know that galaxies have ellipticities and position angles that vary with radius, and so, at some level, the models studied so far incorrectly represent the galaxy light profile. However, at this point, the kinematic uncertainties likely dominate the results, as opposed to assumption biases. One can see this by comparing the inclined models for M32 (Verolme et al. 2002) with the edge-on model of van der Marel et al. (1998). Even there, the difference in the black hole mass is only at the 10% level, and the change in internal orbital structure is even less. Since none of the other black holes are as well measured as M32's (most have uncertainties around 30%), this suggests that the assumption biases will not have a great effect. Yet, once the quality of the data improves, we will have to consider more general models. In fact, triaxial models have already been studied by Verolme et al. (2004). We know that kinematically distinct cores are common in galaxies, and, therefore, axisymmetric models will clearly not provide the best representation. Verolme et al. extend the orbit-based models to include a triaxial distribution function and have successfully reproduced the complicated kinematic structure of NGC 4365. An important step now would be to run both an axisymmetric and triaxial model on the same galaxy to see if any significant differences arise.

The ultimate analysis method includes running an *N*-body model for each galaxy. We know that at some level there is no galaxy that has perfect symmetry. The question then becomes how significant are the errors one makes when running a model that has some symmetry (spherical, axisymmetric, or triaxial) to an asymmetric galaxy. At least for the black hole mass, the errors are not large. Kormendy (2004) summarizes the changes in black hole mass over time and with different dynamical modeling sophistication. He finds that the change in black hole mass, at least for a few well-studied galaxies, is not very large, considering the enormous change in both data and modeling. The black hole masses measured by Magorrian et al. (1998) using low-quality ground-based data and 2-integral models measured black hole masses to within a factor of 2–3 of the presently accepted values. However, the intrinsic scatter of the $M_\bullet - \sigma$ correlation is consistent with zero, and at most 30% (Tremaine et al. 2002). Furthermore, the correlation of black hole mass with other galaxy properties—concentration index (Graham et al. 2001; Graham 2004) and total mass (Magorrian et al. 1998; McLure & Dunlop 2002)—have a low scatter as well. The fact that the scatter in these correlations is already so low implies that the systematic uncertainties are not terribly measured; otherwise, we would not be able to detect these correlations. In order to better study these correlations, we must have better determined black hole masses, and therefore we must improve the analysis techniques. Hopefully, we will not have to measure black hole masses to much better than 10% to answer the scientifically important questions, since going beyond that will be a challenge in terms of both observations and analysis.

References

Adams, F. C., Graff, D. S., Mbonye, M., & Richstone, D. O. 2003, ApJ, 591, 125

Barnes, J., & Hernquist, L. 1992, ARA&A, 30, 705

Binney, J. 1978, MNRAS, 183, 501

Binney, J., Davies, R. L., & Illingworth, G. D. 1990, ApJ, 361, 78

Binney, J., & Mamon, G. A. 1982, MNRAS, 200, 361

Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)

Bower, G. A., et al. 2001, ApJ, 550, 75

Cappellari, M., Verolme, E. K., van der Marel, R. P., Verdoes Kleijn, G. A., Illingworth, G. D., Franx, M., Carollo, C. M., & de Zeeuw, P. T. 2002, ApJ, 578, 787

Cretton, N., de Zeeuw, P. T., van der Marel, R. P., & Rix, H.-W. 1999, ApJS, 124, 383

Cretton, N., Rix, H.-W., & de Zeeuw, P.T. 2000, ApJ, 536, 319

Cretton, N., & van den Bosch, F. C. 1999, ApJ, 514, 704
Davies, R. L., Efstathiou, G., Fall, S. M., Illingworth, G., & Schechter, P. L. 1983, ApJ, 266, 41
de Zeeuw, P. T. 1985, MNRAS, 216, 273
——. 2004, in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), in press
de Zeeuw, P. T., et al. 2002, MNRAS, 329, 513
Dejonghe, H., & de Zeeuw, P. T. 1988, ApJ, 333, 90
Dubinski, J. 1998, ApJ, 502, 141
Emsellem, E., Dejonghe, H., & Bacon, R. 1999, MNRAS, 303, 495
Faber, S. M., et al. 1997, AJ, 114, 1771
Gebhardt, K., et al. 2000a, AJ, 119, 1157
——. 2000b, ApJ, 539, L13
——. 2003, ApJ, 583, 92
Gebhardt, K., & Fischer, P. 1995, AJ, 109, 209
Gebhardt, K., Richstone, D., Ajhar, E. A., Kormendy, J., Dressler, A., Faber, S. M., Grillmair, C., & Tremaine, S. 1996, AJ, 112, 105
Graham, A. W. 2004, in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Pasadena: Carnegie Observatories, http://www.ociw.edu/ociw/symposia/series/symposium1/proceedings.html)
Graham, A. W., Erwin, P., Caon, N., & Trujillo, I. 2001, ApJ, 563, L11
Khairul Alam, S. M., & Ryden, B. S. 2002, ApJ, 570, 610
King, I. R. 1966, AJ, 71, 64
Kormendy, J. 1993, in The Nearest Active Galaxies, ed. J. Beckman, L. Colina, & H. Netzer (Madrid: Consejo Superior de Investigaciones Científicas), 197
——. 2004, in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), in press
Kormendy, J., & Richstone, D. 1995, ARA&A, 33, 581
Kuzmin, G. G., & Kutuzov, S. A. 1962, Bull. Abastumani Ap. Obs., 27, 82
Lauer, T. R. 2004, in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), in press
Levison, H., & Richstone, D. O. 1987, ApJ, 314, L476
Magorrian, J., et al. 1998, AJ, 115, 2285
McLure, R. J., & Dunlop, J. S. 2002, MNRAS, 331, 795
Merritt, D. 1985, AJ, 90, 1027
Merritt, D., & Gebhardt, K. 1994, in Clusters of Galaxies, Proceedings of the XXIXth Rencontre de Moriond, ed. F. Duret, A. Mazure, & J. Tran Thanh Van (Singapore: Editions Frontiéres), 11
Merritt, D., Meylan, G., & Mayor, M. 1997, AJ, 114, 1074
Meza, A., Navarro, J. F., Steinmetz, M., & Eke, V. R. 2003, ApJ, 590, 619
Michie, R. W. 1963, MNRAS, 125, 127
Milosavljević, M., & Merritt, D. 2001, ApJ, 563, 34
Nakano, T., & Makino, J. 1999, ApJ, 510, 155
Osipkov, L. P. 1979, Pisma Ast. Zh., 5, 77
Pinkney, J., et al. 2004, ApJ, in press
Quinlan, G., & Hernquist, L. 1997, NewA, 2, 533
Quinlan, G., Hernquist, L., & Sigurdsson, S. 1995, ApJ, 440, 554
Ravindranath, S., Ho, L. C., Peng, C. Y., Filippenko, A. V., & Sargent, W. L. W. 2001, AJ, 122, 653
Richstone, D. O. 1984, ApJ, 281, 100
Richstone, D. O., & Tremaine, S. 1984, ApJ, 286, 27
——. 1988, ApJ, 327, 82
Rix, H.-W., de Zeeuw, P. T., Carollo, C. M. C., Cretton, N., & van der Marel, R. P. 1997, ApJ, 488, 702
Samland, M., & Gerhard, O. 2003, A&A, 399, 961
Schwarzschild, M. 1979, ApJ, 232, 236
Sigurdsson, S. 2004, in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), in press
Statler, T. S. 1987, ApJ, 321, 113
Toomre, A. 1982, ApJ, 259, 535
Tremaine, S., et al. 2002, ApJ, 574, 740

Tremblay, B., & Merritt, D. 1995, AJ, 110, 1039
Valluri, M., Merritt, D., & Emsellem, E. 2004, ApJ, submitted (astro-ph/0210379)
van der Marel, R. P. 1991, MNRAS, 253, 710
van der Marel, R. P., Cretton, N., de Zeeuw, P. T., & Rix, H.-W. 1998, ApJ, 493, 613
Verolme, E. K., et al. 2002, MNRAS, 335, 517
——. 2004, MNRAS, submitted (astro-ph/0301070)
Zhao, H. S. 1996, MNRAS, 283, 149