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Gas-dynamical Processes in Dense Nuclei

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Abstract

We review recent progress in simulating star formation on a small scale and ask if the insights gained can be applied to the formation of supermassive black holes in dense nuclei. Topics discussed include the origin of the Salpeter initial mass function (IMF), the role of feedback, and a proposal for how the characteristic stellar mass might vary in a cosmological context. We also present surprising evidence that the black hole-bulge mass relation is entirely consistent with such objects representing the tail of a Salpeter IMF in these systems, but remain agnostic about the interpretation of this result.

1.1 Introduction

This review describes some recent work that explores a number of hydrodynamical processes in the context of star formation. Many of the simulations involved are sufficiently recent that their implications have scarcely been digested in their original context, so one should be properly sceptical about the attempts made here to map out some tentative applications to the formation of supermassive black holes (SMBHs).

It is currently unclear how much star formation and SMBH formation have in common. In a general sense, one can see that both processes involve a generic question—given a mass of self-gravitating gas, how does this distribute itself among Jeans unstable objects? Does the bulk of the mass collapse into a single mass concentration or are objects produced over a large dynamic range in mass scales? If the latter occurs, are there processes subsequent to the initial fragmentation process (such as coalescence and competitive accretion) which might drive one toward the former scenario? Recently, and more controversially, Larson (2002, 2003) has hinted that SMBH formation may be "no more than" the high-mass end of the star formation process, involving "no more than" a scaled-up version of the processes operating at the stellar-mass scale. Here we remain agnostic on this issue, although we briefly examine if there is any empirical evidence that this is *not* the case.

Most of the simulations described here have modeled rather small-scale systems with masses in the range of hundreds to thousands of $M_{\odot}$, as typify star-forming complexes such as Taurus and the Orion Nebula cluster. There are two reasons why such environments have been explored rather than the massive/low-metallicity regions that might provide a more suitable environment for producing SMBHs. First, these local regions produce stars with a reasonably well-calibrated mass-luminosity relation and thus the mass spectrum of the objects produced is quite well constrained observationally (see, e.g., Hillenbrand & Carpenter

2000; Luhman et al. 2000). The second reason is a technical one. In nearby star-forming regions, the typical stellar mass is around a solar mass, whereas the minimum ("opacity limit") mass for fragmentation is about a factor 100 lower than this. In the case of star formation with primordial abundances, however, the typical stellar mass scale is around $1000 M_{\odot}$, whereas the opacity limit is 6 orders of magnitude lower than this (Schneider et al. 2002). Thus, a much larger dynamic range of mass scales has to be captured in order to model the primordial problem. Therefore, notwithstanding the seminal simulations of Population III star formation by Bromm, Coppi, & Larson (1999, 2002) and Abel, Bryan, & Norman (2000), the first fully resolved calculations of *ensembles* of stars have been in the context of Population I systems (Bate, Bonnell, & Bromm 2002a,b, hereafter BBB).

We start by briefly describing these first fully resolved simulations of Population I clustered star formation. Since these simulations are at the borderline of what is computationally feasible, it is impossible to explore a wide range of initial conditions, or to treat large ensembles of stars. Consequently, many of the further conclusions about the nature of cluster formation, and, in particular, an analytical understanding of the process, have been derived from more idealized, computationally cheaper, calculations. Before indulging in extrapolations to star formation in different environments, therefore, we should be reasonably assured that the calculations are roughly compatible with observations in the Population I case.

The computational expense in these simulations derives from the necessity of resolving the minimum Jeans mass that can develop in the calculations. These calculations do not treat radiative transfer in the gas, but instead model its thermal evolution by a piecewise polytropic equation of state derived from detailed radiative transfer calculations of spherically symmetric collapse (Larson 1969; Masunaga & Inutsuka 2000). This equation of state (involving a transition from isothermal to adiabatic behavior at the point that dust opacity renders the collapse optically thick) imparts the gas with a minimum Jeans mass corresponding to conditions at this transition. This mass (known as the opacity limit; Low & Lynden-Bell 1976; Rees 1976) is a few Jupiter masses. The BBB calculations (Fig. 1.1) employ smoothed particle hydrodynamics (SPH: see Monaghan & Gingold 1983; Benz et al. 1990), in which all fluid quantities are evaluated by weighted averaging over ensembles of 50 particles. Therefore, in order to resolve all structures that can conceivably collapse in the calculation, one must assign at least ~ 100 particles to the mass scale of the opacity limit (see Bate & Burkert 1997; Whitworth 1998). Evidently, the number of particles required for the simulation of an astronomically interesting cluster of stars is enormous. The BBB simulations employ 3.5 million particles to model $50 M_{\odot}$ of gas and require 95000 CPU hours on a SGI Origin 3800 supercomputer.

The BBB calculations are currently unique in that they resolve down to the opacity limit, but are in other respects the natural extension of a number of pioneering simulations that have modeled the evolution of turbulent molecular gas (Vázquez-Semadeni, Ballesteros-Paredes, & Rodriguez 1997; Mac Low et al. 1998; Stone, Ostriker, & Gammie 1998; Padoan & Nordlund 1999; Klessen, Heitsch, & Mac Low 2000). Following these authors, the BBB calculations start with initially homogeneous gas that is endowed with a divergence-free random Gaussian velocity field with power spectrum $P(k) \propto k^{-4}$, this choice being motivated by the desire to reproduce the observed size-linewidth relation in molecular clouds (Larson 1981). (Note that, contrary to some of the calculations cited above, the turbulence is not driven, but instead decays on roughly a cloud-crossing time scale.) The initial supersonic turbulent velocity field produces a complex web of sheets and filaments of shock-compressed

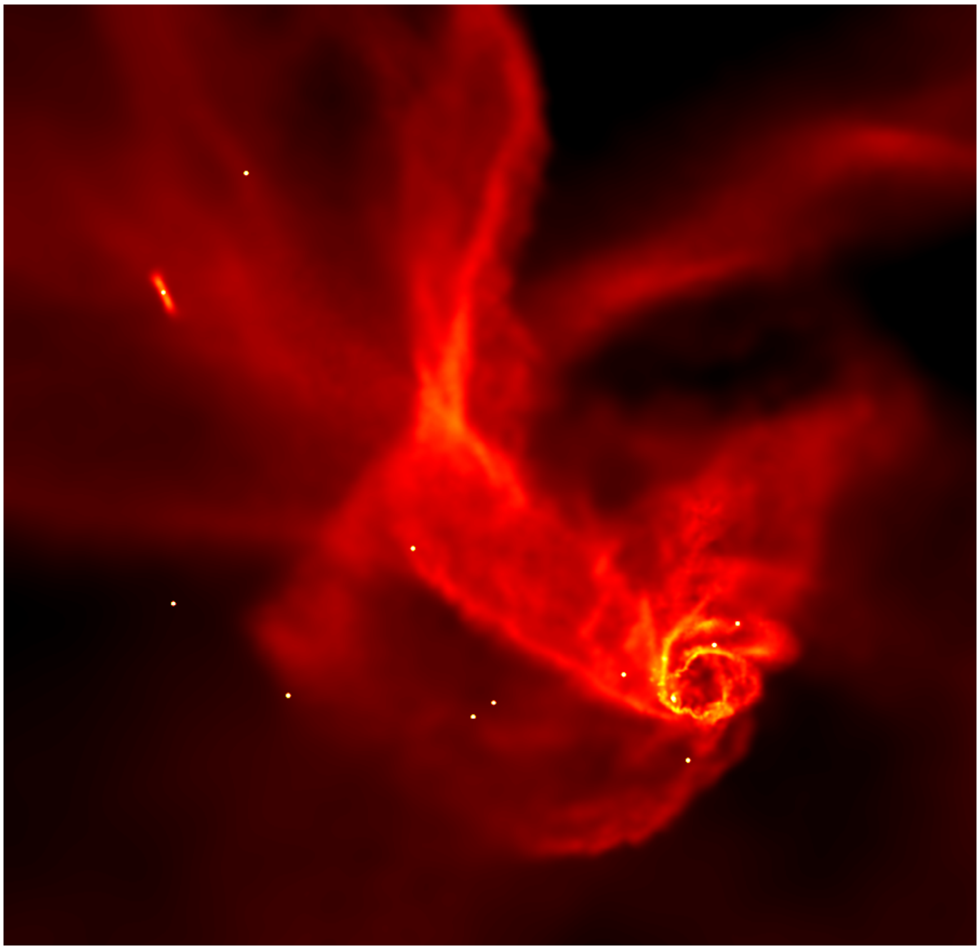

Fig. 1.1. Snapshot from BBB simulations depicting the small clusters that form at the intersection of gas filaments. Note the slingshot ejection of brown dwarf (and edge-on disk) in upper left. The box is 5100 AU on a side. (Courtesy of Matthew Bate.)

gas: stars form, initially at the opacity limit, through fragmentation of both these filaments and of circumstellar disks, and thereafter grow in mass by accretion. This accretion is *competitive* in the sense that protostars accrete from a common gas reservoir, and their orbital history determines their relative success in acquiring mass. Thus, whereas some stars remain close to the center of mass of the ensemble, and accrete rapidly, others suffer slingshot encounters and are ejected, being thereafter deprived of accretable material. Thus, the chaotic orbital dynamics and the consequently diverse accretion histories of the various stars result in a large dynamic range of final masses.

The results of these simulations (i.e., the mass spectrum of stars and brown dwarfs, the kinematics of young stars, the presence of circumstellar disks, and the statistics of binary and multiple systems) can be compared in detail with observations. At a crude level, the results appear to be generally consistent with observations (see Bate, Bonnell, & Bromm

2003), although there are a number of detailed areas (particularly regarding the separations and stability of multiple systems) where further work is required.

Having assured ourselves that these calculations are not grossly incompatible with observations, we shall use this "success" to justify many of the subsequent discussions. This "success" does not mean that these calculations incorporate all the relevant physics. For example, they omit magnetic fields because of the well-known difficulty of incorporating magnetic fields in SPH (see Phillips & Monaghan 1985). Magnetic fields are clearly present in molecular clouds, and a number of authors have constructed models in which these fields play a central role in regulating star formation (e.g., Shu, Adams, & Lizano 1987; McKee 1989). It is now known, however, following Mac Low et al. (1998) and Stone et al. (1998), that magnetic fields are ineffective in one of the roles originally assigned to them, namely the "support" of molecular clouds against global collapse due to the magnetic cushioning of shocks. These simulations (see also Ostriker, Stone, & Gammie 2001) imply that magnetic fields can support molecular clouds only in regions where the mass to flux ratio is subcritical (i.e., roughly at the point where the magnetic energy density exceeds the gravitational potential energy density of the cloud). Regions of the cloud that are not threaded by such strong fields instead collapse on a dynamical time scale (as in the BBB simulations), although fragmentation may be modified by the local magnetic field structure. Clearly, the incorporation of magnetic fields in cluster formation simulations remains an important goal. For now, noting also that magnetic fields may be less important in the early Universe than in Population I systems, we do not consider them further.

1.2 The IMF and the Maximum Object Mass

The IMF in the local Universe is well constrained observationally (see, e.g., Kroupa 2002 and references therein) and is characterized by a power law at high masses such that the number of stars in the mass range m to $m+dm$ is $\propto m^{-\alpha}dm$ with $\alpha = 2.35$ (Salpeter 1955). At masses below around a solar mass, the slope flattens somewhat to give α in the range 1 to 1.5 (Kroupa, Tout, & Gilmore 1990). The mean mass of a stellar population is chiefly set by the upper (lower) limit of the power law if α is $< (>) 2$. It thus follows that the transition in α from > 2 to < 2 at around a solar mass imprints the local IMF with a characteristic mass scale of around $1\,M_\odot$.

Over the years, fashions concerning the universality of the IMF have come and gone, and the types of departure from universality discussed have likewise varied, with bimodal IMFs and IMFs with variable upper slope having once been popular parameterizations (see, e.g., Güsten & Mezger 1982; Larson 1986, 1998a, b). As the evidence has gathered that the IMF in the local Universe is surprisingly uniform (at least with regard to its upper power law, which may be the only part that is observationally accessible in more distant systems), a popular view is that any variation is likely to be accommodated in a variable value of the characteristic mass (Larson 1998a, b, 2003; Clarke & Bromm 2003). The justification for this view is that a power law is by its nature scale free and may reflect the operation of processes that are themselves blind to scale. A characteristic stellar mass, by contrast, is a dimensional quantity that is likely to depend on some property of the star-forming system.

In an IMF in which the upper power law has a slope ~ 2.35, massive stars are rare and do not dominate the overall mass budget of the system. If there are no *physical constraints* on the upper end of this power law, then the maximum mass in a given stellar system is simply set by the statistics of randomly drawing a finite number of stars from such an IMF.

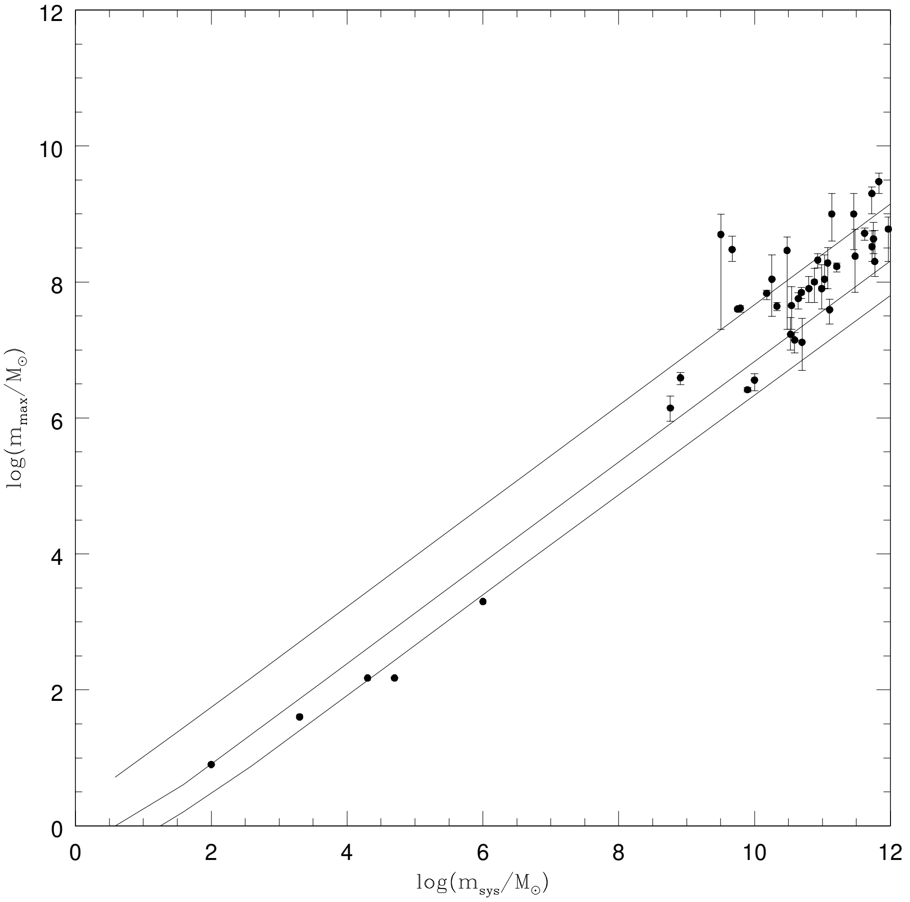

Fig. 1.2. The 95th, 50th, and 5th centiles of the distribution of maximum object mass as a function of system mass, assuming objects are selected from a Salpeter IMF for $M > 0.1\,M_{\odot}$. See text for the source of observational data.

Evidently, in a populous system, one is more likely to draw objects from the upper reaches of the IMF than in a small-N system, and, given an IMF, one may readily construct the probability density function of the most massive star as a function of N. The IMF, however, also defines a mean stellar mass $\bar{m}$, such that the system mass is $M_{\rm sys} = N\bar{m}$. In Figure 1.2 we depict the 5th, 50th and 95th centile of the probability density function of the most massive object as a function of $M_{\rm sys}$, assuming a Salpeter IMF ($\alpha = 2.35$) for object mass $> 0.1\,M_{\odot}$. The data points correspond to massive stars in local star-forming regions (Ophiuchus, Orion Nebula cluster, and 30 Doradus) taken from the compilation in Larson (2003) for objects less than $\sim 200\,M_{\odot}$, the inferred black hole in M15 (Gerssen et al. 2002) with mass $\sim 2000\,M_{\odot}$ and, for the objects more massive than $\sim 10^6\,M_{\odot}$, the masses of SMBHs as a function of bulge mass derived from Kormendy & Gebhardt (2001).

The surprising aspect of Figure 1.2 is that the data are so consistent with the "random drawing" hypothesis over about 10 orders of magnitude in system mass! This consistency is unsurprising in the low-mass (stellar) regime, since the IMF has been derived from star counts in such regions. [The fact that the data points in this region suggest a rather shallower slope than predicted by the Salpeter model, may provide some evidence that the IMF slope is somewhat steeper at high masses than the Salpeter value, as suggested by Larson (1982, 2003), although the small number of data points do not make the case compelling.] What is surprising—and quite possibly fortuitous—is the excellent fit to the SMBH data, with the slope, normalization and scatter in the data all being reproduced by this most simple model.

What significance, if any, should one attach to this result? "Random drawing" is of course not a physical description of either the formation of massive stars or SMBHs, but provides a way of quantifying the statistical consequences of a given spectrum of object mass. We have seen that the existence of such a spectrum (combined with a lack of any physical limitations on the masses produced) implies a relation between maximum object mass and the total mass of the reservoir involved. Therefore any physical model that produces such a spectrum must have a way of communicating the total reservoir mass to the region forming the most massive object (even though this object comprises a small fraction of the total reservoir mass). This is reasonable in observed star-forming complexes where the dynamical time is less than the lifetime of the stars concerned and where there is thus time for the reservoir to communicate with the central region where the most massive object is generally formed (see discussion in § 1.4 and § 1.5 of how accretion into the central regions controls the history of competitive accretion and coalescence in these regions and dictates the maximum mass of stars produced). In the case of SMBHs forming in the bulges of galaxies, the dynamical time scales are considerably longer, but on the other hand, the accreting lifetime of the SMBH is indefinite. It is not clear that we can reject the analogy between the two processes on these grounds.

If such a correlation tells us something about analogies between SMBH formation and massive star formation, would this be incompatible with current wisdom (Haehnelt & Kauffmann 2000; Kauffmann & Haehnelt 2000) about the hierarchical assembly of galactic bulges and the build up of black holes through accretion and mergers? The answer here is "probably not." A process that produced a Salpeter-like IMF extending up to supermassive objects could either function in the monolithic collapse of a bulge or, equally, in its constituent progenitors. In the latter case, it would not make much difference to the $M_\bullet - M_{\rm bulge}$ relation if the final black hole was the most massive object in the progenitor halos or the merged sum of such objects (because, for a Salpeter IMF, the mass contained in equal decades of object mass is roughly constant, and therefore one could merge a large dynamic range of hole masses and only increase the final hole mass by a factor of a few). One might even ascribe the fact that the data points are somewhat above the predicted centiles in Figure 1.2 to this effect, although this is probably over-interpretation of the data.

In summary, then, the data on SMBH mass versus bulge mass are, somewhat surprisingly, not incompatible with SMBHs being formed as a massive tail of a mass function extending down to stellar masses. We have not found any obvious physical reasons why a communication between the total reservoir mass and the mass of the maximum object should not occur in both the stellar and the supermassive regime, although the arguments about the extent of the analogy are not at all well developed. Are there other observational grounds for ruling out this conjecture? One implication is that in massive (galactic-scale) systems, there should be a number of objects intermediate in mass between the most massive (central) black hole and objects of stellar mass. Although objects with $M > 10^6 M_\odot$ are likely to have spiralled in and merged with the central black hole, this still leaves a large dynamic range of objects with masses in the range $10^2 - 10^6 M_\odot$ that would be free floating in the galaxy. A number of authors have considered the detectability, or otherwise, of a population of black holes in the halo of the Galaxy (e.g., Carr & Sakellariadou 1999), but were attempting to explain the bulk of the dark matter by such objects. In the current case, the massive black holes would be a minority contributor to the mass budget of the galaxy—considerably less than that contained in stars—and so it is not clear that such a population could be ruled out.

1.3 The Origin of the Characteristic Stellar Mass

We noted above that the IMF in the local Universe is imprinted with a characteristic mass of around a solar mass. An obvious question regards the physics that controls this value, and how one might expect this to vary as a function of environment. Are there, for example, regions of the Universe where very massive stars form not as part of the Salpeter upper-mass tail of a distribution but instead represent the characteristic mass of that population?

In local star-forming regions, this mass scale may readily be identified with the Jeans mass corresponding to dense ($n \approx 10^5$ cm^{-3}), cool ($T \approx 10$ K) molecular gas in star-forming cores (Benson & Myers 1993). This temperature is set by thermal balance between heating (by cosmic rays or UV dust heating) and cooling (by molecular line radiation and infrared dust cooling; see Whitworth, Boffin, & Francis 1998). The density is apparently controlled by a state of rough balance between the thermal pressure in the cores and the mean internal pressure of bulk motions ("turbulence") within the parent giant molecular cloud (GMC; Larson 1996, 1998a). This conjecture is supported by hydrodynamic simulations of turbulent clouds. For example, Padoan, Nordlund, & Jones 1997 (see also Padoan & Nordlund 2002) have shown that driven supersonic turbulence in an isothermal medium produces a characteristic density at which the thermal pressure roughly balances the turbulent pressure in the cloud and have argued that the characteristic stellar mass is the corresponding isothermal Jeans mass. Similar conclusions may be derived from the BBB simulations described in § 1.1, where the turbulence is not driven and where collapse and fragmentation are fully resolved. Here ,too, the mean stellar mass appears to be set by the gas temperature and the turbulent pressure of the initial conditions.

What, then, sets the "turbulent pressure" in star-forming clouds? Two factors may be relevant. First, the attainment of the low temperatures required for star formation requires that molecular coolants are self-shielded against photodissociation by the ambient UV field (e.g., van Dishoeck & Black 1988). This imposes a minimum column for molecular clouds of around 3×10^{21} cm^{-2}. In a self-gravitating system in which the flow velocities are comparable to their free-fall values, the pressure depends on the column density according to $P \sim G\Sigma^2$ and thus the self-shielding requirement implies a minimum viable pressure of around 2×10^4 K cm^{-3}. On the other hand, the ambient pressure of the ISM sets a lower limit to the mean internal pressure (local GMCs are somewhat self-gravitating and thus the internal pressure of molecular clouds exceeds the ambient pressure by around an order of magnitude). The pressure within GMCs (or, equivalently, their column densities) does, however, appear to scale with the ambient interstellar pressure, in that the pressures within GMCs

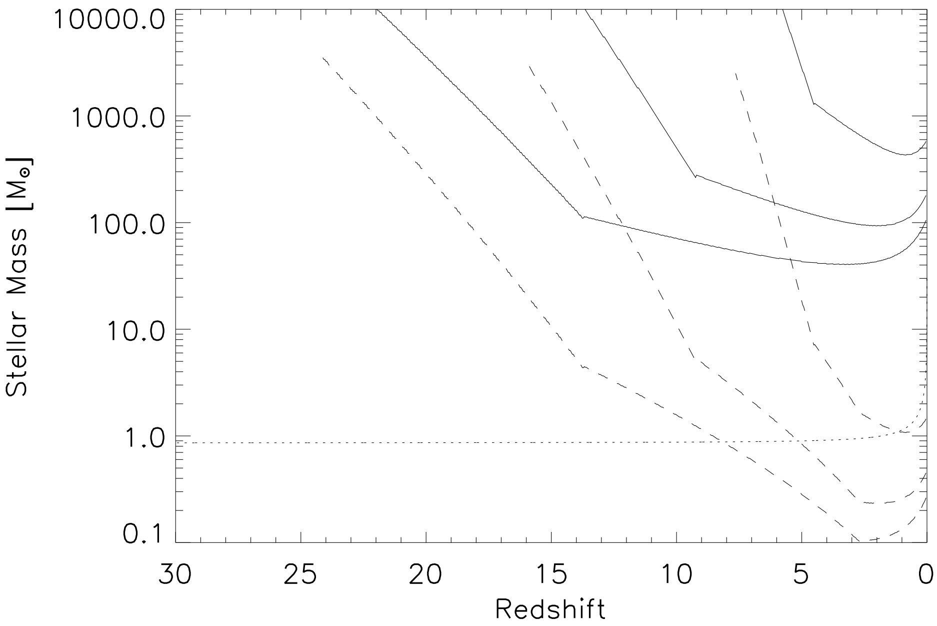

Fig. 1.3. The dependence of characteristic stellar mass on redshift of virialization. (From Clarke & Bromm 2003; see text for details.)

in the Galactic Center are higher than those in local clouds (Sanders, Scoville, & Solomon 1985).

Recently, Clarke & Bromm (2003) have extended these arguments so as to estimate the characteristic mass scales associated with the bursts of star formation accompanying the assembly of proto-galaxies. Again, the characteristic mass scale is identified with the Jeans mass at given temperature and pressure. The temperature is jointly set by the appropriate cooling agent ($\sim$ 10 K for CO line cooling, $\sim$ 200 K for H_2 cooling in primordial gas) and the floor set by the temperature of the cosmic microwave background. The pressure, in this model, is set by the weight of overlying baryons in collapsing halos of given mass and virialization redshift. This latter choice is motivated by the results of hydrodynamical simulations of star formation in low-mass halos, which show that gas that has been heated to the virial temperature collapses nearly isobarically (Bromm et al. 2001). The problem thus reduces to determining the mean pressure of the baryons at the point of virialization. The results of this exercise are illustrated in Figure 1.3, where the solid and dashed lines represent lines of constant overdensity (1, 2, and 3σ from top to bottom) in a ΛCDM cosmology, with cooling provided by H_2 and CO, respectively. The characteristic stellar mass rises toward high z, due both to the increasing temperature of the cosmic microwave background and the decreasing pressure associated with the lower mass halos virializing at high redshift.

This simple prescription roughly reproduces both the near-solar values of the characteristic mass, $M_{\rm char}$, inferred in the local Universe and the high masses found in numerical simulations of Population III star formation. To see if it is consistent with all observational constraints, it has to be included in hierarchical merger models, and the variation of the functional form of the IMF with redshift must be specified. If we assume (see § 1.2), that the characteristic stellar mass may vary with cosmic conditions but that the functional form of the IMF should remain constant, then the sorts of observations that can test the model are those that link the relative numbers of stars above and below the characteristic mass scale. For example, Hernandez & Ferrara (2002) have argued that the relative paucity of very metal poor stars in the halo of the Milky Way requires a rather high characteristic mass scale (around $10 M_\odot$) in low-mass halos ($10^8 - 10^9 M_\odot$) at redshifts $5-10$, in good agreement with the model. On the other hand, the element *ratios* in low metallicity objects are expected to tend to the Type II plateau value (Gibson 1998; Wyse et al. 2002) in all cases that $M_{\rm char}$ is below the minimum progenitor mass of a Type II supernova, since in that case all the objects providing the enrichment would lie on the Salpeter tail of the IMF. Likewise, population synthesis models for Lyman-break galaxies (which imply a Salpeter IMF for stars down to early B spectral type; Pettini et al. 2002) only require that $M_{\rm char}$ is less than $15-20 M_\odot$.

An important implication of this model for the production of very massive black holes (and possibly SMBHs) is that even in the case of H_2 cooling, the characteristic stellar mass falls with decreasing redshift due purely to the higher pressures involved in the higher mass halos that collapse at later epochs. We see from Figure 1.3 that for z in the range $10-15$ the characteristic stellar mass for gas in $2-3\sigma$ peaks falls to a few hundred solar masses. Stars in this mass range are able to undergo pair-production supernovae, implying that very low metallicity conditions are unlikely to prevail at more recent epochs. We therefore conclude that it is very unlikely that an IMF biased toward very high masses could persist at redshifts less than $10-15$. According to Schneider et al. (2002), this would be sufficient to account for the density of SMBHs in nearby galaxies (assuming that very massive black holes formed in this mode eventually merged into SMBHs that settled in galactic nuclei) but would not produce enough remnant black holes to account for the bulk of baryonic dark matter.

1.4 The Origin of the Upper End of the Power-law IMF

A wide variety of scale-free processes have been invoked to explain the power-law nature of the upper IMF, including coalescence (Silk & Takahashi 1979; Murray & Lin 1996), fragmentation (Silk 1978), and the result of feedback (Silk 1995; Adams & Fatuzzo 1996). In the present section we focus on a process that produces such an IMF in the BBB simulations described in § 1.1, namely *competitive accretion*. In this case, final stellar masses are not set predominantly by their mass scale when they first become Jeans unstable but instead by their subsequent accretion history from a distributed gaseous reservoir which they share with other accreting protostars. (Note that the success of competitive accretion in providing such a power law does not preclude the importance of other processes for the production of the observed IMF. It does imply, however, that even if these other processes are important, one cannot use the IMF argument to justify their necessity.)

The term "competitive accretion" was first coined by Zinnecker (1992) when he showed that a power-law IMF is readily generated from Bondi-Hoyle (1944) accretion on to seed masses with a small spread in initial masses. The argument may be simply reproduced: for an object of instantaneous mass M, the Bondi-Hoyle accretion rate (assuming the object is moving at constant speed with respect to an infinite, homogeneous background) scales as $\dot{M} \propto M^2$. Thus, at time t

$$\frac{1}{M} = \frac{1}{M_{\rm init}} - kt,$$

for some constant k. This implies that an initial spread in masses, $dM_{\rm init}$, maps on to a range of final masses (at some time t) according to

$$dM = \left(\frac{M}{M_{\rm init}}\right)^2 dM_{\rm init}.$$

Now, if the number of seeds remains constant as they grow, we thus deduce that the number of seeds per unit mass range must decline as M^{-2}. Thus, in its simplest form, competitive accretion generates an IMF of slope -2, tantalizingly close to the Salpeter slope of -2.35.

Evidently, a real cluster environment is rather different from the idealized model considered by Zinnecker (for example, the gas is likely to be inhomogeneous or self-gravitating and the stars' velocity with respect to the gas will not remain constant). The role of competitive accretion in clusters has been explored by "plum pudding" models (Bonnell et al. 1997, 2001a,b; Delgado-Donate, Clarke, & Bate 2003), which represent an intermediate state of realism. In these numerical (SPH) models, a large number of gravitational "sinks" are distributed within an initially homogeneous self-gravitating gas cloud, with most of the mass being initially in the distributed gas. The evolution of the gas/sink system is then followed hydrodynamically, with accretion of gas onto the sinks (stars) being handled by the sink-particle formulation of Bate, Bonnell, & Price (1995).

Analysis of such simulations reveals two regimes in which stars may accrete the bulk of their mass. Initially, the whole system collapses (because it contains a large number of Jeans masses) and the stars collapse with the gas. As the stars are in this case essentially passengers in the gas flow, the relative velocity, $v_{\rm rel}$, between the stars and gas is low, and hence the Bondi-Hoyle radius ($GM/v_{\rm rel}^2$, which sets the cross section for Bondi-Hoyle accretion) is correspondingly very high. In such circumstances, gas approaching at this large impact parameter would be prevented from accreting by the tidal field of the parent cluster, and it is found that the accretion cross section is set by the tidal (Roche) radius of each sink. At late times, and in the central regions of the cluster, the stars start to dominate the potential locally and can undergo violent relaxation, resulting in a virialized stellar core. Gas continues to flow into this central region, but now the relative velocity between the inflowing gas and the virialized stars is high ($\sim$ the free-fall velocity). In this case, the Bondi-Hoyle radius is less than the tidal radius and now sets the accretion cross section.

These two accretion regimes give rise to two different regimes in the resulting IMF. The Bondi-Hoyle accretion that occurs in the virialized core gives rise to an IMF of slope around -2 (see Bonnell et al. 2001a,b) for a discussion of why the slopes in the simulations are rather steeper than this, in even better agreement with the Salpeter value). In the other case, where the accretion cross section is set by the tidal radius one may show analytically that the resulting IMF has slope -1.5 (Bonnell et al. 2001b). This picture is confirmed when one calculates for each star whether it acquired the bulk of its mass in an environment where the local mass density is mainly contributed by the gas or in a virialized core where the stars are locally dominant. It is found (Fig. 1.4) that stars that gain their mass mainly in the gas-dominated regime are lower mass stars, which follow an IMF slope of -1.5, whereas higher mass stars gain most of their mass in the star-dominated, virialized core of

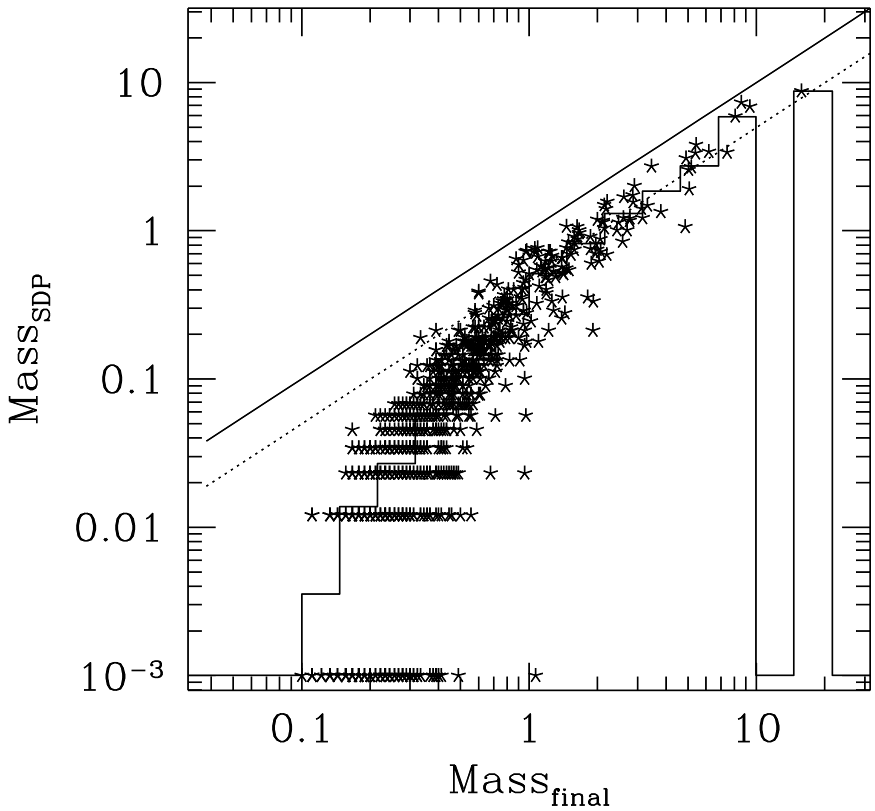

Fig. 1.4. The mass each star acquired in the star-dominated regime (see text) is plotted against its final mass (both in arbitrary units). The histogram plots binned averages of the same quantities and the solid and dashed lines depict the case that 100% and 50% of the final mass is acquired in this regime. The IMF in these simulations shows a transition to a steeper (Salpeter-like) IMF at a mass $\sim$ 1 in these units. (From Bonnell et al. 2001b.)

the cluster, and follow an IMF with slope ~ -2. [Note: the position of the "knee" in the composite IMF is set purely by the average mass per star, since (see § 1.2), the mean mass is always close to the location where the IMF makes a transition between a slope greater or less than -2. In these "plum pudding" models, the mean final stellar mass is put into the initial conditions. In a more realistic model, where the seeds result from Jeans instability in an inhomogeneous and dynamic medium, one might expect the mean stellar mass to be set instead by the considerations set out in § 1.3—i.e., by the minimum gas temperature and dynamical pressure in the parent cloud. The BBB simulations (see § 1.1) appear to corroborate this conjecture.]

1.5 The Highest Mass Stars

In the simulations of populous cluster formation by Bonnell et al. (see § 1.4), the most massive stars form in the center of the cluster. This outcome is consistent with the widely observed tendency of massive stars to be located in cluster cores (Clarke, Bonnell, & Hillenbrand 2000). Since, however, two-body relaxation also leads to the same outcome, one can only *prove* that primordial mass segregation is required in a few well-observed and very young clusters (see the *N*-body simulations of Bonnell & Davies 1998 for such a demonstration in the Orion Nebula cluster).

In cluster formation simulations, stars near the cluster center accrete their local gas reservoir and hence undergo violent relaxation ahead of the rest of the cluster, thereby forming a kinematically distinct core in a state of rough virial equilibrium. Gas, however, continues to flow into this central region. If the core's dynamical time scale becomes less than the gas inflow time scale, then the core subsequently evolves adiabatically and its radius shrinks strongly with accreted mass ($R \propto M^{-3}$). Thus, the accretion of relatively modest quantities of gas produces a dramatic shrinkage of the central core and a very steep ($\propto M^9$) increase of the number density of stars in the core.

Bonnell, Bate, & Zinnecker (1998) first suggested that this shrinkage could ultimately lead to stellar collisions and that the possible merger of collision products could provide a promising mechanism for the formation of the most massive stars. The conditions required for mergers are extreme (stellar number densities of at least 10^8 pc^{-3}) and exceed the observed stellar densities in the densest young clusters by at least 3 orders of magnitude. It is therefore necessary to posit that the high-density phase in which collisions occur is very brief, and that the cluster core rapidly reexpands. Bonnell et al. invoked the destructive effects of feedback (e.g., from ionizing radiation) to expel gas from the core and reexpand it to more reasonable stellar densities.

There seems little doubt that simulations show an inexorable runaway toward high-mass objects in the cores of populous clusters, due to the combined effects of accretion and mergers. The runaway is favored in clusters where the gas flow has a single focus (usually the cluster's center of mass) and where the virialized core is relatively populous. In the BBB simulations, by contrast, the flow (at least on the time scale followed in the calculation) is focused on multiple centers (set up by the pattern of shocks following the dissipation of the initial turbulence), and each center contains only a small number of stars. Under these conditions, the small stellar systems tend to break up (by dynamical encounters and slingshot ejections) before they are driven by accretion to the very high densities required for mergers. From this we conclude (rather unsurprisingly) that the formation of very massive objects is likely to be favored by the monolithic collapse of massive systems.

The simulations, however, neglect feedback and assume that stars stick when they collide. This latter assumption is probably reasonable at the rather low velocities (tens of km s^{-1}) of these cluster cores (M. Davies, private communication). Note that it is found (Bonnell & Bate 2002) that mergers rarely result from a hyperbolic flyby, but instead follow the hardening of a close massive binary by continued accretion and by perturbations from loosely bound companion stars. An anticipated feature of this scenario is that massive stars should frequently be found in close binary pairs with other massive stars, in good agreement with observations (Mermilliod & García2001).

Because effective feedback mechanisms are associated with more massive stars, feedback breaks the scale-free nature of the processes described above and allows one to consider

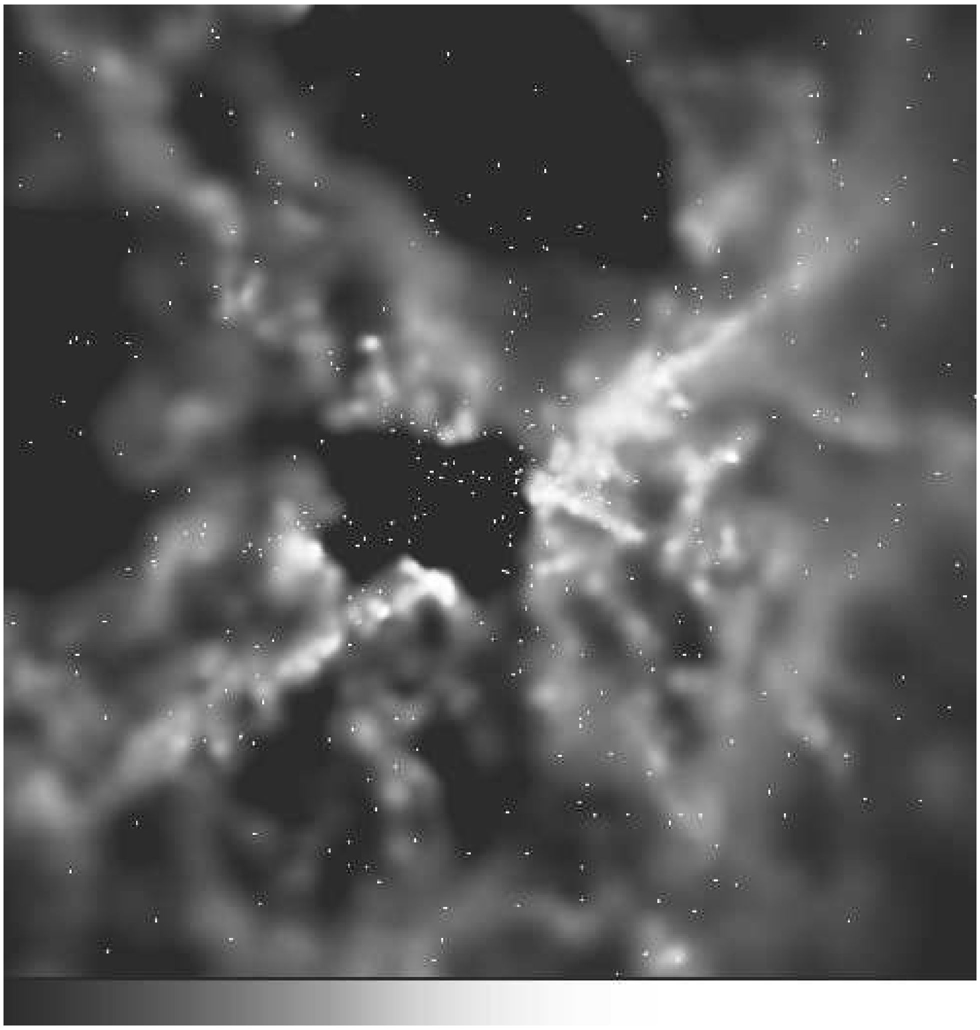

Fig. 1.5. The effect of ionizing radiation from a very massive ($220 M_\odot$) star on a protocluster. Gas column density is plotted in the central 1.4 pc$\times$1.4 pc of a cluster containing $\sim 500 M_\odot$ of gas. The central gas density prior to the switch on of the ionizing source exceeded 10^8 cm^{-3}. (Courtesy of Jim Dale.)

physical (as opposed to statistical) limits on the maximum stellar mass. Although the effects of supernovae on young clusters can be dramatic, the time that must elapse before supernovae explode (2 Myr even in the case of the most massive stars) is comfortably longer than the dynamical time scales of observed clusters (and indeed *much* longer than the dynamical time scales in the extremely dense cores posited in the merger models). The remaining relevant feedback mechanisms are stellar winds, ionization, and the effect of radiation pressure on dust, and each of these will both oppose accretion onto individual stars and expel gas from the cluster core, thus reducing the incidence of stellar collisions.

Simulations are currently being developed that model the effect of radiation pressure on dust during Bondi-Hoyle accretion onto massive stars (Edgar et al., in prep.), while others are treating the effect of ionizing radiation on the gas dynamics in the cluster core (Dale et al., in prep.; see also Kessel-Deynet & Burkert 2000). It appears that feedback limits the range of initial cluster parameters (cloud mass and radius and number of stars forming) that can undergo this runaway to very high densities and consequent stellar collisions. If the local gas density and escape velocity when massive stars form is insufficiently high, ionizing radiation can effectively heat and expel gas from the core (Fig. 1.5) and the runaway is reversed. At higher densities, however, ionizing radiation may be confined to very compact regions around the stars, and the effect on the cluster dynamics may be small.

1.6 Conclusions

In this paper we have reviewed recent progress in modeling the hydrodynamics of star cluster formation and have asked if there is any relevance of the processes discussed to the formation of SMBHs. If there is a link, it is clearly in the area of IMF predictions. Can one scale up the parameters considered here so as to predict the incidence of very massive objects, say stars of hundreds of solar masses, that may collapse to form very massive black holes? To be more provocative, can one even apply these arguments to the formation of objects that have never been stars, such as SMBHs? And, if not, why not? Several themes have run through this paper, and although they may be applied in combination, they are not inextricably linked. Thus, one might accept the arguments about the statistics of drawing objects from a scale-free distribution, but favor a different physical origin for this scale-free IMF than the one advocated here. Likewise, the speculations about how the characteristic mass for star formation may vary with cosmic conditions are independent of a specific model for the upper IMF. Below, we briefly summarize the main themes.

(1) In systems where the distribution of resulting masses follows a power law at high masses, the *maximum object mass* is a simple function of this power-law slope, the lower limit of this power law, and the total mass of the system. This just represents the obvious statistical truth that one is likely to pick a higher mass from the distribution if one starts with a larger mass reservoir (and hence a larger number of random drawings). We have examined if there is any empirical evidence that one cannot apply this argument to systems with a wide range of masses. To our surprise, we have found that if one simply assumes that all systems follow a Salpeter IMF (to arbitrarily high masses), one predicts a distribution of maximum object mass as a function of system mass that is in remarkable agreement with observations over 10 orders of magnitude in system mass. In particular, the masses of SMBHs as a function of host bulge mass and the masses of the putative black holes in globular clusters are in excellent agreement with the hypothesis that the SMBHs represent the high-mass tail of the Salpeter IMF in these systems. We remain agnostic about the significance of this result, but note that observations provide no empirical disproof of this simple hypothesis.

(2) We furthermore speculate that whereas the slope of the upper IMF may be a universal constant, produced, for example, by the scale-free processes described in (3) below, the *characteristic stellar mass* (i.e., the point at which the IMF flattens to a slope less than –2), *may* be a function of environment and epoch. We present a simple model that predicts the characteristic mass for star formation during the assembly of galaxies as a function of halo mass and redshift. This model successfully reproduces both the stellar mass scale associated with simulations of Population III star formation, as well as star formation in the

local Universe. We discuss the implications of this model for the production of very massive black holes at high redshift.

(3) We discuss a model for the IMF suggested by simulations of cluster formation. Competitive accretion plays a central role in these simulations: the final mass of an object depends on its history of competing with neighboring protostars from a common gas reservoir. The upper (Salpeter) tail to the IMF is produced by Bondi-Hoyle accretion in the virialized core of the cluster. We furthermore discuss how such a core may enter a regime of adiabatically invariant shrinkage in response to gas inflow, and how this may lead to the very high densities required for stellar collisions. If these collisions lead to coalescence, the uppermost end of the IMF may be populated by the products of stellar mergers in short-lived, ultra-dense cluster cores.

(4) We briefly discuss how feedback may modify the scale-free nature of the processes described in (3), and may thus undermine the assumptions behind the exercise described in (1). Ongoing simulations of the role of ionizing radiation and radiation pressure on dust on the dynamics of cluster cores suggest that the avoidance of strong feedback (and hence the possibility of growing to very high masses) may put particular constraints on the parameters of the parent cluster.

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