Carnegie Observatories Astrophysics Series, Vol. 1:
Coevolution of Black Holes and Galaxies, 2004
ed. L. C. Ho (Pasadena: Carnegie Observatories,
http://www.ociw.edu/ociw/symposia/series/symposium1/proceedings.html)

Dense Gas-Star Systems: Evolution of Supermassive Stars

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Abstract

In the 60s and 70s super-massive central objects (from now onwards SMOs) were thought to be the main source of active galactic nuclei (AGNs) characteristics (luminosities of $L \approx 10^{12} L_{\odot}$). The release of gravitational binding energy by the accretion of material on to an SMO in the range of $10^7 - 10^9 M_{\odot}$ has been suggested to be the primary powerhouse (Lynden-Bell 1969). That rather exotic idea in early time has become common sense nowadays. Not only our own galaxy harbours a few million-solar mass black hole (Genzel 2001) but also many of other non-active galaxies show kinematic and gas-dynamic evidence of these objects (Magorrian et al. 1998) The concept of central super-massive stars (SMSs henceforth) ($\mathcal{M} \geq 5 \times 10^4 M_{\odot}$, where $\mathcal{M}$ is the mass of the SMS) embedded in dense stellar systems was suggested as a possible explanation for high- energy emissions phenomena occurring in AGNs and quasars (Vilkoviski 1976, Hara 1978), such as X-ray emissions (Bahcall and Ostriker, 1975). SMSs and super-massive black holes (SMBHs) are two possibilities to explain the nature of SMOs, and SMSs may be an intermediate step towards the formation of SMBHs (Rees 1984). In this paper we give the equations that describe the dynamics of such a dense star-gas system which are the basis for the code that will be used in a prochain future to simulate this scenario. We also briefly draw the mathematical fundamentals of the code.

1.1 The gaseous model

To go from stationary to dynamical models we use a gaseous model of star clusters in its anisotropic version. It is based on the basic assumptions that: the system can be described by a one particle distribution function, the secular evolution is dominated by the cumulative effect of small angle deflections with small impact parameters (Fokker-Planck approximation, good for large N-particle systems) and that the effect of the two-body relaxation can be modelled by a local heat flux equation with an appropriately tailored conductivity.

The first assumption justifies a kinetic equation of the Boltzmann type with the inclusion of a collisional term of the Fokker-Planck (FP) type:

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \dot{v_r} \frac{\partial f}{\partial v_r} + \dot{v_\theta} \frac{\partial f}{\partial v_\theta} + \dot{v_\varphi} \frac{\partial f}{\partial v_\varphi} = \left(\frac{\delta f}{\delta t} \right)_{FP} \tag{1.1}$$

In spherical symmetry polar coordinates r, θ, ϕ are used and t denotes the time. The vector

$\mathbf{v} = (v_i), i = r, \theta, \phi$ denotes the velocity in a local Cartesian coordinate system at the spatial point r, θ, ϕ. The distribution function f is a function of, r, t, v_r, $v_\theta^2 + v_\phi^2$ only due to spherical symmetry. By multiplication of the Fokker-Planck equation (1.1) with various powers of the velocity components we get up to second order a set of moment equations which is equivalent to gas-dynamical equations coupled with Poisson's equation: A mass equation, a continuity equation, an Euler equation (force), radial and tangential energy equations. The system of equations is closed by a phenomenological heat flux equation for the flux of radial and tangential r.m.s. kinetic energy, both in radial direction.

1.2 Interaction terms for the star component

We now introduce the interaction terms to be added to right hand of the star component equations.

1.2.1 Equation of continuity

In the paper by Langbein et al. (1990) they derive the interaction terms to be added to the basic equations of the gaseous model. According to them, the star continuity equation is no longer

$$\frac{\partial \rho_*}{\partial \mathrm{t}} + \frac{1}{\mathrm{r}^2}\frac{\partial}{\partial \mathrm{r}}(\mathrm{r}^2 \rho_* \mathrm{u}_*) = 0, \tag{1.2}$$

but

$$\frac{\partial \rho_*}{\partial \mathrm{t}} + \frac{1}{\mathrm{r}^2}\frac{\partial}{\partial \mathrm{r}}(\mathrm{r}^2 \rho_* \mathrm{u}_*) = \left(\frac{\delta \rho_*}{\delta \mathrm{t}}\right)_{\mathrm{coll}} + \left(\frac{\delta \rho_*}{\delta \mathrm{t}}\right)_{\mathrm{lc}}; \tag{1.3}$$

where the right-hand term reflects the time variation of the star's density due to stars interactions (i.e. due to the calculation of the mean rate of gas production by stars collisions) and loss-cone (stars plunging onto the central object).

If $f(v_{\mathrm{rel}})$ is the stellar distribution of relative velocities, then the mean rate of gas production by stellar collisions is

$$\left(\frac{\delta \rho_*}{\delta \mathrm{t}}\right)_{\mathrm{coll}} = -\int_{|v_{\mathrm{rel}}| > \sigma_{\mathrm{coll}}} \frac{\rho_* f_c(v_{\mathrm{rel}})}{t_{\mathrm{coll}}} f(v_{\mathrm{rel}}) d^3 v_{\mathrm{rel}} \tag{1.4}$$

$f(v_{\mathrm{rel}})$ is a Schwarzschild-Boltzmann distribution,

$$f(v_{\mathrm{rel}}) = \frac{1}{2\pi^{3/2}\sigma_r \sigma_t^2} \cdot \exp\left(-\frac{(v_{\mathrm{rel,r}} - u_*)^2}{4\sigma_r^2} - \frac{v_{\mathrm{rel,t}}^2}{2\sigma_t^2}\right) \tag{1.5}$$

As regards f_c, it is the relative fraction of mass liberated per stellar collision into the gaseous medium. Under certain assumptions given in the initial work of Spitzer & Saslaw (1966), we can calculate it as an average over all impact parameters resulting in $r_{\mathrm{min}} < 2r_*$ and as a function of the relative velocity at infinity of the two colliding stars, v_{rel}. Langbein et al. (1990) approximate their result by

$$f_c(v_{\mathrm{rel}}) = \begin{cases} \left(1 + q_{\mathrm{coll}}\sqrt{\sigma_{\mathrm{coll}}/v_{\mathrm{rel}}}\right)^{-1} & v_{\mathrm{rel}} > \sigma_{\mathrm{coll}} \\ 0 & v_{\mathrm{rel}} < \sigma_{\mathrm{coll}}, \end{cases}$$

with $q_{\mathrm{coll}} = 100$. So, we have that

$$f_c(v_{\rm rel}) = \begin{cases} 0.01 & \sigma_{\rm coll} = v_{\rm rel} \\ 0 & \sigma_{\rm coll} > v_{\rm rel}, \end{cases}$$

$t_{\rm coll}$ is defined as the mean time which has passed when the number of stars within a volume $V = \Sigma \cdot v_{\rm rel} \cdot \triangle t$ is one, where $v_{\rm rel}$ is the relative velocity at infinity of two colliding stars. Computed for an average distance of closest approach $\bar{r}_{\rm min} = \frac{2}{3} r_*$, this time is

$$n_* V(t_{\rm coll}) = 1 = n_* \Sigma v_{\rm rel} t_{\rm coll}. \tag{1.6}$$

And so,

$$t_{\rm coll} = \frac{m_*}{\rho_* \Sigma \sigma_{\rm rel}}, \tag{1.7}$$

with

$$\Sigma = \pi \bar{r}^2_{\rm min} \left(1 + \frac{2Gm_*}{\bar{r}_{\rm min} \sigma^2_{\rm rel}} \right); \tag{1.8}$$

$\sigma^2_{\rm rel} = 2\sigma^2_*$ is the stellar velocity dispersion and Σ a collisional cross section with gravitational focusing.

The first interaction term is *

$$\left(\frac{\delta \rho_*}{\delta \rm t} \right)_{\rm coll} = -\rho_* \frac{\rm f_c}{\rm t_{coll}} \left[1 - {\rm erf} \left(\frac{\sigma_{\rm coll}}{\sqrt{6} \sigma_{\rm r}} \right) \right] \left[1 - {\rm erf} \left(\frac{\sigma_{\rm coll}}{\sqrt{6} \sigma_{\rm t}} \right) \right]^2 \tag{1.9}$$

which, for simplification, we re-call like this

$$\left(\frac{\delta \rho_*}{\delta \rm t} \right)_{\rm coll} \equiv -\rho_* {\rm X_{coll}}. \tag{1.10}$$

Since the evolution of the system that we are studying can be regarded as stationary, we introduce for each equation the “logarithmic variables” in order to study the evolution at long-term. In the other hand, if the system happens to have quick changes, we should use the “non-logarithmic” version of the equations. For this reason we will write at the end of each subsection the equation in terms of the logarithmic variables.

In the case of the equation of continuity, we develop it and divide it by ρ_* because we are looking for the logarithm of the stars density, $\partial \ln \rho_* / \partial t = (1/\rho_*) \partial \rho_* / \partial t$. The result is:

$$\frac{\partial \ln \rho_*}{\partial t} + \frac{\partial u_*}{\partial r} + u_* \frac{\partial \ln \rho_*}{\partial r} + \frac{2u_*}{r} = \frac{1}{\rho_*} \left(\frac{\delta \rho_*}{\delta \rm t} \right)_{\rm coll} + \frac{1}{\rho_*} \left(\frac{\delta \rho_*}{\delta \rm t} \right)_{\rm lc} \tag{1.11}$$

1.2.2 Momentum balance

The second equation has the following star interaction terms:

$$\frac{\partial u_*}{\partial t} + u_* \frac{\partial u_*}{\partial r} + \frac{GM_r}{r^2} + \frac{1}{\rho_*} \frac{\partial p_r}{\partial r} + 2 \frac{p_r - p_t}{\rho_* r} = \left(\frac{\delta u_*}{\delta t} \right)_{\rm drag} \tag{1.12}$$

The interaction term is due to the decelerating force at which stars that move inside the gas are subject to. Explicitely, it is

$$\left(\frac{\delta u_*}{\delta t} \right)_{\rm drag} = -X_{\rm drag} \frac{1}{\rho_*} (u_* - u_g) \tag{1.13}$$

* In the paper there are two typos in the equation, the correct signs and factors are given here.

where we have introduced the following definition:

$$X_{\rm drag} \equiv -C_D \frac{\pi r_*^2}{m_*} \rho_* \rho_g \sigma_{\rm tot}, \tag{1.14}$$

with $\sigma_{\rm tot}^2 = \sigma_r^2 + \sigma_t^2 + (u_* - u_g)^2$

To the end of the calculation of the logarithmic variable version of the equation, we multiply by $\rho_* r/p_r$:

$$\frac{\rho_* r}{p_r}\left(\frac{\partial u_*}{\partial t} + u_*\right) + \frac{GM_r}{rp_r}\rho_* + \frac{\partial \ln p_r}{\partial \ln r} + 2(1 - \frac{p_t}{p_r}) = -X_{\rm drag}\frac{r}{p_r}(u_* - u_g) \tag{1.15}$$

1.2.3 Radial energy equation

As regards the last but one equation, the interaction terms are:

$$\frac{\partial p_r}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_* p_r) + 2p_r\frac{\partial u_*}{\partial r} + \frac{4}{5}\frac{(2p_r - p_t)}{t_{\rm relax}} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) - \frac{2F_t}{r} = \tag{1.16}$$

$$\left(\frac{\delta p_r}{\delta t}\right)_{\rm drag} + \left(\frac{\delta p_r}{\delta t}\right)_{\rm coll}, \tag{1.17}$$

where

$$\left(\frac{\delta p_r}{\delta t}\right)_{\rm drag} = -2X_{\rm drag}\sigma_r^2, \ \left(\frac{\delta p_r}{\delta t}\right)_{\rm coll} = -X_{\rm coll}\rho_* \tilde{\sigma_r}^2 \epsilon. \tag{1.18}$$

In order to determine ϵ we introduce the ratio k of kinetic energy of the remaining mass after the encounter over its initial value (before the encounter); k is a measure of the inelasticity of the collision: for $k = 1$ we have the minimal inelasticity, just the kinetic energy of the liberated mass fraction is dissipated, whereas if $k < 1$ a surplus amount of stellar kinetic energy is dissipated during the collision (tidal interactions and excitation of stellar oscillations). If we calculate the energy loss in the stellar system per unit volume as a function of k we obtain

$$\epsilon = f_c^{-1}[1 - k(1 - f_c)]. \tag{1.19}$$

We divide by p_r so that we get the logarithmic variable version of the equation. We make also the following substitution:

$$\begin{aligned} F_r &= 3p_r v_r \\ F_t &= 2p_t v_t \end{aligned} \tag{1.20}$$

The resulting equation is

$$\frac{\partial \ln p_r}{\partial t} + (u_* + 3v_r)\frac{\partial \ln p_r}{\partial r} + 3\left(\frac{\partial u_*}{\partial r} + \frac{\partial v_r}{\partial r}\right) + \frac{2}{r}\left(u_* + 3v_r - 2v_t\frac{p_t}{p_r}\right) + \frac{4}{5}\frac{2 - \frac{p_t}{p_r}}{t_{\rm relax}} = \frac{1}{p_r}\left(\frac{\delta p_r}{\delta t}\right)_{\rm drag} + \frac{1}{p_r}\left(\frac{\delta p_r}{\delta t}\right)_{\rm coll} \tag{1.21}$$

1.2.4 Tangential energy equation

To conclude the set of equations of the star component with the interaction terms, we have the following equation:

$$\frac{\partial p_t}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_* p_t)+2\frac{p_t u_*}{r}-\frac{4}{5}\frac{(2p_r-p_t)}{t_{\rm relax}}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_t)+\frac{2F_t}{r}= \tag{1.22}$$

$$\left(\frac{\delta p_t}{\delta t}\right)_{\rm drag}+\left(\frac{\delta p_t}{\delta t}\right)_{\rm coll}, \tag{1.23}$$

where

$$\left(\frac{\delta p_t}{\delta t}\right)_{\rm drag}=-2X_{\rm drag}\sigma_t^2,\ \left(\frac{\delta p_t}{\delta t}\right)_{\rm coll}=-X_{\rm coll}\rho_*\tilde{\sigma_t}^2\epsilon. \tag{1.24}$$

We follow the same path like in the last case and so we get the following logarithmic variable equation:

$$\begin{aligned}\frac{\partial \ln p_t}{\partial t}+(u_*+2v_t)\frac{\partial \ln p_t}{\partial r}+\frac{\partial}{\partial r}(u_*+2v_t)+\frac{4}{r}(u_*+2v_t)-\\ \frac{4}{5}\frac{2\frac{p_r}{p_t}-1}{t_{\rm relax}}=\frac{1}{p_t}\left(\frac{\delta p_t}{\delta t}\right)_{\rm drag}+\frac{1}{p_t}\left(\frac{\delta p_t}{\delta t}\right)_{\rm coll}\end{aligned} \tag{1.25}$$

1.3 The gaseous component and the interaction terms

In this section we give the set of equations corresponding to the gaseous component as for their right hand interaction terms.

1.3.1 Equation of continuity

For the SMS the equation of continuity looks as follows:

$$\frac{\partial \rho_g}{\partial t}+\frac{1}{\mathrm{r}^2}\frac{\partial}{\partial r}(\mathrm{r}^2\rho_g u_g)=\left(\frac{\delta \rho_g}{\delta t}\right)_{\rm coll} \tag{1.26}$$

where, for the mass conservation, we have that, obviously,

$$\left(\frac{\delta \rho_g}{\delta t}\right)_{\rm coll}=-\left(\frac{\delta \rho_*}{\delta t}\right)_{\rm coll} \tag{1.27}$$

We follow the same procedure as for the star continuity equation to get the equation in terms of the logarithmic variables:

$$\frac{\partial \ln \rho_g}{\partial t}+\frac{\partial u_g}{\partial r}+u_g\frac{\partial \ln \rho_g}{\partial r}+\frac{2u_g}{r}=\frac{1}{\rho_g}\left(\frac{\delta \rho_g}{\delta \mathrm{t}}\right)_{\rm coll} \tag{1.28}$$

The interaction term is in this case

$$\frac{1}{\rho_g}\left(\frac{\delta \rho_g}{\delta \mathrm{t}}\right)=\frac{1}{\rho_g}\left(-\frac{\delta \rho_*}{\delta \mathrm{t}}\right)=-\frac{\rho_*}{\rho_g}X_{\rm coll} \tag{1.29}$$

1.3.2 Momentum balance

We modify equation number (2.9) of Langbein et al. (1990) in the following way:

$$\frac{\partial(\rho_g u_g)}{\partial t} = u_g \frac{\partial \rho_g}{\partial t} + \rho_g \frac{\partial u_g}{\partial t}; \tag{1.30}$$

we substitute this equality in their equation, divide by ρ_g (u_g is the variable in our code) and make use of the equation of continuity for the gas component. Thus, we get the following expression:

$$\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial r} + \frac{GM_r}{r^2} + \frac{1}{\rho_g}\frac{\partial p_r}{\partial r} - \frac{4\pi}{c}\kappa_{\rm ext} H = \left(\frac{\delta u_g}{\delta t}\right)_{\rm coll} \tag{1.31}$$

To get the interaction term we use the mass and momentum conservation:

$$\left(\frac{\delta \rho_g}{\delta t}\right)_{\rm coll} + \left(\frac{\delta \rho_*}{\delta t}\right)_{\rm coll} = 0 \tag{1.32}$$

$$\left(\frac{\delta(\rho_g u_g)}{\delta t}\right)_{\rm coll} + \left(\frac{\delta(\rho_* u_*)}{\delta t}\right)_{\rm coll} = 0. \tag{1.33}$$

We know that

$$\left(\frac{\delta u_*}{\delta t}\right)_{\rm coll} = 0, \tag{1.34}$$

thus,

$$\left(\frac{\delta(\rho_g u_g)}{\delta t}\right)_{\rm coll} = u_* \rho_* X_{\rm coll} = \rho_g \left(\frac{\delta u_g}{\delta t}\right)_{\rm coll} + u_g X_{\rm coll} \rho_* . \tag{1.35}$$

Therefore, the resulting interaction term is

$$\left(\frac{\delta u_g}{\delta t}\right)_{\rm coll} = \frac{\rho_*}{\rho_g} X_{\rm coll}(u_* - u_g) \tag{1.36}$$

In the case of the stellar system

$$F = \frac{1}{2}(F_r + F_t) = \frac{5}{2}\rho_* v_* \tag{1.37}$$

By analogy, we now introduce $F_{\rm rad}$ in this way

$$\frac{F_{\rm rad}}{4\pi} = H = \frac{5}{2} p_g v_g, \tag{1.38}$$

where v_g is per gas particle.

$$v_g = \frac{2}{5}\frac{H}{p_g} \tag{1.39}$$

As means to write the equation in its "logarithmic variable version", we multiply the equation by $\rho_g r/p_g$, as we did for the corresponding momentum balance star equation and replace H by $5/2 p_g v_g$,

$$\frac{\rho_g r}{p_g}\left(\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial r}\right) + \frac{GM_r}{r p_g}\rho_g + \frac{\partial \ln p_g}{\partial \ln r} - \frac{5}{2}\frac{\kappa_{\rm ext}}{c}\rho_g r v_g = \frac{r}{p_g}\rho_* X_{\rm coll}(u_* - u_g) \tag{1.40}$$

1.3.3 Radiation transfer

We get the radiation transfer equations by re-writing the frequency-integrated moment equations from Amaro-Seoane et al. (2003): We divide their first equation by J and multiply the second one by $2c/(5p_g v_g)$,

$$\frac{1}{c}\frac{\partial \ln J}{\partial t}+\frac{5}{2J}\frac{\partial}{\partial r}(v_g p_g)+\frac{5}{Jr}p_g v_g-\frac{3f_{\rm Edd}-1}{cr}u_g-(1+f_{\rm Edd})\frac{1}{c}\frac{\partial \ln \rho_g}{\partial t}=\frac{\kappa_{\rm abs}}{J}(B-J) \tag{1.41}$$

$$\frac{\partial \ln v_g}{\partial t}+\frac{\partial \ln p_g}{\partial t}+\frac{2c}{5}\frac{1}{p_g v_g}\frac{\partial (J f_{\rm Edd})}{\partial r}+\frac{2c}{5}\frac{3f_{\rm Edd}-1}{r p_g v_g}J-\frac{2u_g}{r}-2\frac{\partial \ln \rho_g}{\partial t}=-c\kappa_{\rm ext}\rho_g \tag{1.42}$$

Where we have substituted $H=5p_g v_g/2$ and $\kappa_{\rm abs}$ and $\kappa_{\rm ext}$ are the absorption and extinction coefficients per unit mass

$$\kappa_{\rm abs}=\frac{\rho_g \Lambda(T)}{B},\ \kappa_{\rm ext}=\rho_g(\kappa_{\rm abs}+\kappa_{\rm scatt}), \tag{1.43}$$

$\Lambda(T)$ is the cooling function, B the Planck function and $\kappa_{\rm scatt}$ the scattering coefficient per unit mass. We have made use of $\partial M_r/\partial r=4\pi^2\rho$, $f_{\rm Edd}=K/J$, and the Kirchhoff's law, $B_\nu=j_\nu/\kappa_\nu$ (j_ν is the emission coefficient).

1.3.4 Thermal energy conservation

It is enlightening to construct an equation for the energy per volume unit $e=(p_r+2p_t)/2$ which, in the case of an isotropic gas ($p_r=p_t$) is $e=3p/2$. For this aim we take, for instance, equation (1.17) and in the term $2p_r\partial u_*/\partial r$ we include now a source for radiation pressure, $2(p_r+p_{\rm rad})\partial u_*/\partial r$ and we divide everything by e so that we get the logarithmic variables. The resulting equation is

$$\frac{\partial \ln e}{\partial t}+(u_g+3v_g)\frac{\partial \ln e}{\partial r}+\frac{2}{r}(u_g+v_g)=\frac{1}{e}\left(\frac{\delta e}{\delta t}\right)_{\rm drag}+\frac{1}{e}\left(\frac{\delta e}{\delta t}\right)_{\rm coll} \tag{1.44}$$

The interaction terms for this equation are

$$\left(\frac{\delta e}{\delta t}\right)_{\rm drag}=X_{\rm drag}(\sigma_r^2+\sigma_t^2+(u_*-u_g)^2) \tag{1.45}$$

$$\left(\frac{\delta e}{\delta t}\right)_{\rm coll}=\frac{1}{2}X_{\rm coll}\rho_*((\sigma_r+\sigma_t)^2\epsilon+(u_*-u_g)^2-\xi\sigma_{\rm coll}^2) \tag{1.46}$$

1.3.5 Mass conservation

The mass conservation is guaranteed by

$$\frac{3}{4\pi}\frac{\partial M_r}{\partial r^3}=\rho_*+\rho_g \tag{1.47}$$

1.4 A mathematical view of the code

Our model has seven dependent variables,

$$(\rho, u, v_r, v_t, p_r, p_t, M) \equiv \mathbf{x}(r,t), \tag{1.48}$$

So we have a set of non-linear, coupled differential equations plus an initial model, which very often is a Plummer's model. To solve our equations we discretise them on a logarithmic radial mesh with typically 200 logarithmically equidistant mesh points, covering radial scales over eight orders of magnitude; e.g. from 100 pc down to 10^{-6} pc, which is enough to resolve the system down to the vicinity of a massive black hole's tidal disruption radius for stars.

An implicit Newton-Raphson-Henyey iterative method is used to solve for the time evolution of our system. Let $\mathbf{F}(x)$ be a column-vector for the seven equations:

$$\mathbf{F}(x) = \begin{pmatrix} F_1(x_1 \ldots x_7) \\ \vdots \\ F_7(x_1 \ldots x_7) \end{pmatrix} \tag{1.49}$$

The solution to the equations is $x_{\rm true}$; therefore,

$$\mathbf{F}(x_{true}) = 0. \tag{1.50}$$

Suppose that $x^{(1)}$ is a close value to the solution $x_{\rm true}$,

$$x^{(1)} = x_{\rm true} + \triangle x; \tag{1.51}$$

thus,

$$\mathbf{F}(x^{(1)}) = \mathbf{F}(x_{\rm true} + \triangle x) = \mathbf{G}(x), \tag{1.52}$$

where $\mathbf{G}(x)$ is an "error function" that contains the difference between the exact value and the approximation. Since we have assumed that $x^{(1)}$ is a close value to $x_{\rm true}$,

$$\mathbf{F}(x_{\rm true} + \triangle x) \approx \mathbf{F}(x_{\rm true}) + \frac{\partial \mathbf{F}}{\partial x} \triangle x = \mathbf{G}(x), \tag{1.53}$$

therefore, since $\mathbf{F}(x_{\rm true}) = 0$,

$$\triangle x = \mathbf{G}(x) \left(\frac{\partial \mathbf{F}}{\partial x} \right)^{-1}, \tag{1.54}$$

where

$$\frac{\partial \mathbf{F}}{\partial x} \equiv \begin{pmatrix} \partial F_1(x_1 \ldots x_7)/\partial x_1 & & \\ & \ddots & \\ & & \partial F_7(x_1 \ldots x_7)/\partial x_7 \end{pmatrix} \tag{1.55}$$

is a 7×7 matrix. We iterate the process until we reach

$$\left| \frac{\triangle x^{(n)}}{x^{(n)}} \right| \leq \varepsilon \sim 10^{-6}, \tag{1.56}$$

where n stands for the n-iteration done. This gives us the termination of the iteration. Our results compare well with other studies using direct solutions of the Fokker-Planck equation or Monte Carlo models (Lightman & Shapiro 1977, Marchant & Shapiro 1980). Note that the Monte Carlo approach has been recently revisited and improved by Freitag & Benz (2001). In contrast to the other models the gaseous model is much more versatile to include all kinds of important other physical effects, such as the dynamics of gas liberated in nuclei by stellar evolution and collisions and its interaction with the stellar component.

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