Carnegie Observatories Astrophysics Series, Vol. 2:
Measuring and Modeling the Universe, 2004
ed. W. L. Freedman (Pasadena: Carnegie Observatories,
http://www.ociw.edu/ociw/symposia/series/symposium2/proceedings.html)

Detecting the Multi-Connectedness of the Universe

B. WILSON
Department of Physics, University of Toronto

C. C. DYER
Department of Astronomy and Astrophysics, University of Toronto

Abstract

We examine the angular two-point correlation function to see if it can be used to detect the multi-connectedness of the universe. This test does not need distance information, therefore it can be used on surveys of objects which do not have redshift information available, thus increasing the amount of data available for use. This test has several drawbacks, for example you cannot determine which particular multi-connected model the universe is nor can you orient it. However, it is a quick and easy test which does detect the multi-connectedness of the universe. One could use this as a first test, then use the more complicated tests which have been previously proposed to get more information about the universe once it was known that the universe is multi-connected.

1.1 Introduction

A cylinder is an example of a multi-connected (MC) plane. The local geometry of both manifolds is identical everywhere, they both have zero intrinsic curvature. The only differences are the global (topological) properties. Since general relativity is a local field theory, MC models are allowed.

MC models are always smaller than the simply-connected (SC) models to which they are locally equivalent. Some MC models are finite in spatial extent unlike their SC counterparts. Quantum cosmology may predict that smaller universes have higher probabilities of being created. If true, this would rule out the spatially infinite models allowing only MC models and the closed Robertson-Walker universe.

MC models are created by taking the SC model, cutting out an appropriate shape and identifying pairs of its border as being the same piece. You can instead imagine folding the shape in many dimensions and gluing the pieces of the border to each other in pairs. These associations can involve twisting. In some cases the amount of twist is constrained by the criteria of orientability. We do not want Klein bottle type universes. Figure 1.1 is an example of a cubic MC manifold with opposite sides matched with a half twist. Photons leave the source "b" and arriver at the observer "O" having wrapped around the cell. Alternatively, the mirror images of "b", having flipped into "p"s, emit photons which arrive at "O". Solid lines represent the "real" path of the photons.

When we look at objects which are different distances away from us, we see them at different ages. This makes identifying multiple images of the same object difficult as we do not know precisely how it evolved. We are also likely to see it from different angles, as

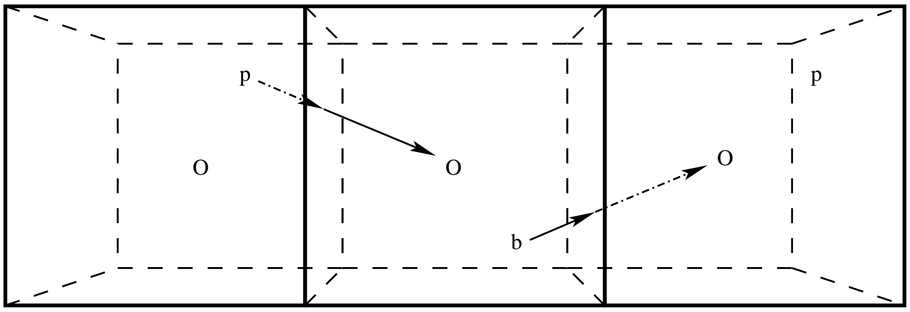

Fig. 1.1. A picture of three cells of a cubic manifold with opposite faces matched with a half twist. Light leaves a source "b" and arrives at an observer "O" after wrapping once around the cell. Another interpretation is that the image "p" emits the light which follows a path to the observer "O".

can be seen from the picture of the cubic MC manifold with twists. It is generally accepted that the best way to observe the effects of an MC universe is via statistical observations of large numbers of similar objects. Figure 1.2 is the data from an MC and an SC model, respectively. It is non-trivial, at least in some cases, to distinguish the two situations.

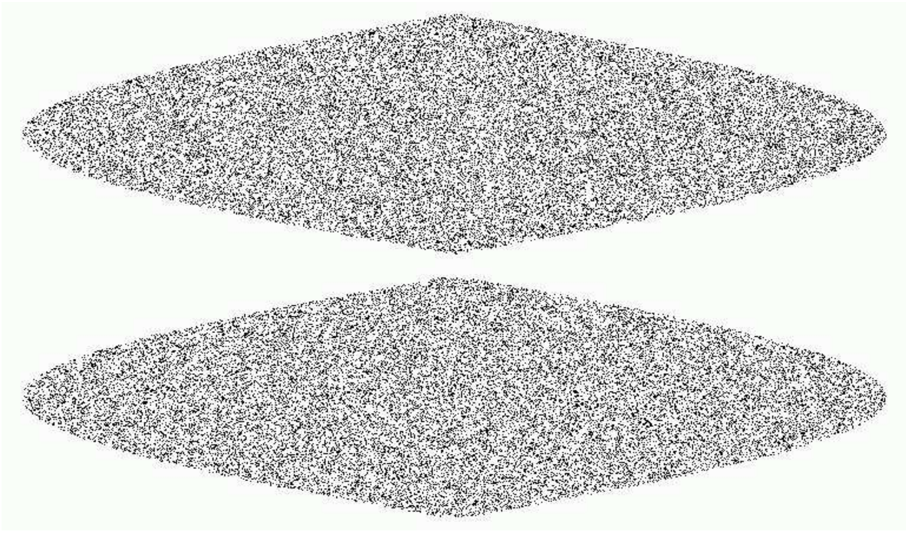

Fig. 1.2. The top is an all-sky map of the data from an MC model. The bottom is that from an SC model. In this case it is difficult to tell the difference just by looking at them.

Our test is a measurement of the 2-point angular correlation function. This is most often used to measure the clustering of objects at small angular scales. The measurement is made by counting the number of pairs of objects which are separated by a given angle (θ). The function should look like $\sin(\theta)$ with Poissonian noise. Deviations from this would indicate that the universe is not SC.

More accurate than the simple 2-point correlation function is the correlation estimator

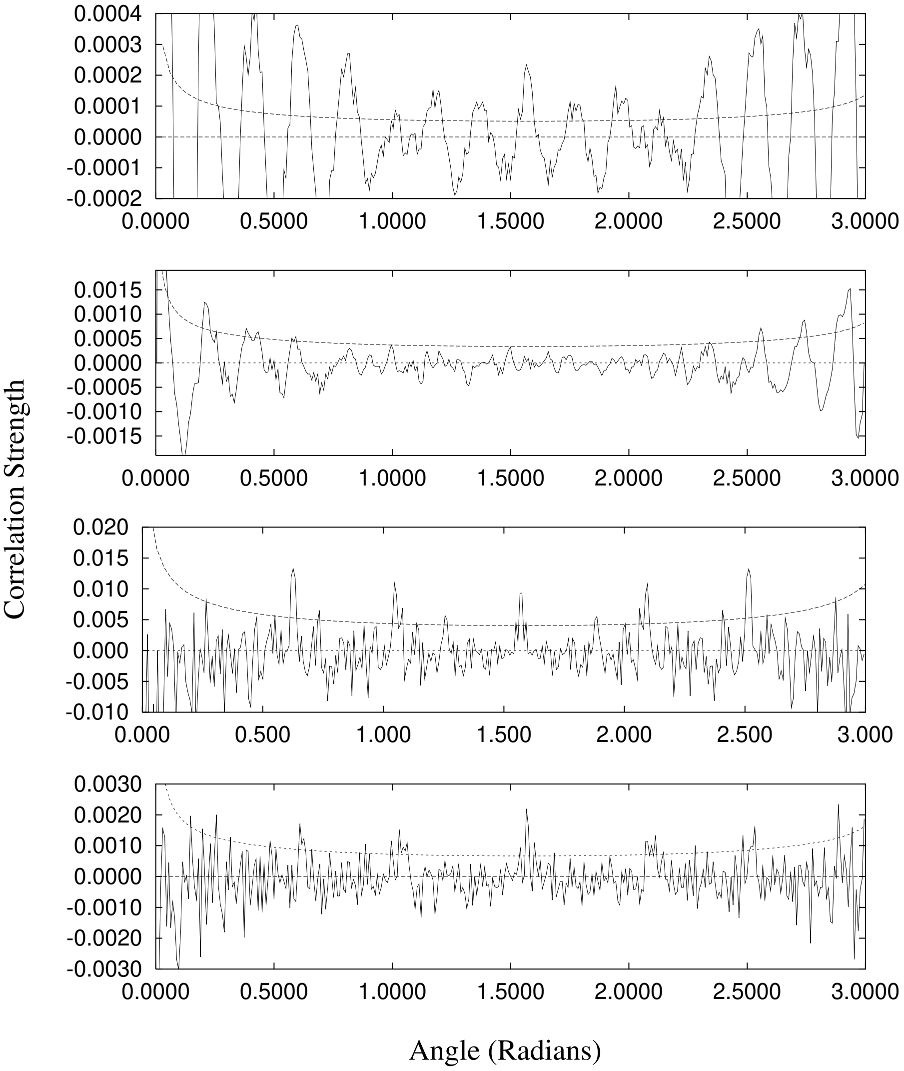

Fig. 1.3. The angular two-point correlation function for, respectively, the cubical model without and with a galactic cut, and the dodecahedral model without and with the galactic cut. The value of the correlation is unimportant, what is important is whether the correlations ia above the smooth line representing 3 standard deviations from a null correlation, assuming Poissonian statistics.

$DD-2DR+RR/RR$ as pointed out by Landy & Szalay (1993). The estimator corrects for issues of geomety such as boundaries. Failing to correct for these issues makes interpreting the 2-point correlation quite difficult.

The galactic plane may interfere with the observations we wish to make. To avoid this, we removed all data within 10° of the equator, orienting the cut in a worst-case scenario. We analyzed both the cases with and without the cut.

1.2 Results

We analyzed 2 models, the cubical MC model with opposite boundaries matched with a half twist, and a dodecahedral model with opposite boundaries matched with a $1/10^{\text{th}}$ twist. The first model corresponds to a flat universe, the second to a closed universe. The first 2 graphs of Figure 1.3 are the correlation functions for the open universe without and with the galactic cut, the other 2 graphs are for the closed universe without and with the galactic cut. The smooth curve corresponds to a positive signal with a strength of 3 standard deviations from a null signal assuming Poissonian statistics. Even with the galactic cut, there is a positive signal for both models.

The source of the signal is the fact that many pairs of cells have their centers separated by the same angle. Thus objects in one cell tend to be separated from images of themselves by roughly the same angle. It is not exact since there are orientation issues; objects not at the center of the cells are offset from the center in different directions with respect to the obeserver depending on which pair of cells one is looking at. This induces some scatter of the signal, making for broader and less high correlation peaks.

Clearly the signal is better with full sky-coverage. That doesn't mean a partial sky survey is useless. We are currently working on partial sky surveys. The preliminary results look promising.

The errors are pretty much limited by statistical errors. The only measurements necessary are the locations of the objects, which can be measured very accurately. Since the bin widths are about half a degree wide, accuracies of a few arcseconds in the location of objects makes the only measurement errors inconsequential.

1.3 Conclusions

As can be seen, even with the worst-case situation of total obscuration due to the plane of our galaxy, the MC universe is observationally different from an SC universe. This conclusion comes with some caveats. First, extra care must be taken to analyze the cases with galactic cuts to reduce the noise. Also, the sample of objects must be visible at sufficiently large distances that a signal is produced. The closed MC model, for example, does not produce a measurable signal unless at least half of the total universe is observable. An all-sky survey would be best, though obscuration by the Milky Way isn't an insurmountable problem. The SLOAN survey may work.

Acknowledgements. This research was funded by operating grants from NSERC Canada and by the University of Toronto.

References

Landy, S. D., & Szalay, A. S. 1993, ApJ, 412, 64