*Carnegie Observatories Astrophysics Series, Vol. 2:*
*Measuring and Modeling the Universe, 2004*
*ed. W. L. Freedman (Pasadena: Carnegie Observatories,*
*http://www.ociw.edu/ociw/symposia/series/symposium2/proceedings.html)*

# Measuring the Time Evolution of the Redshift-Distortion Parameter $\beta$

H. NISHIOKA
*Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan*

**Abstract**

We examine the detectability of the time evolution and the geometric distortion of clustering from the galaxy redshift surveys like the 2dFGRS and SDSS. We present the realistic estimator to infer the cosmological parameters and the derivative of the bias parameter $d\ln b(z)/dz$ at present, based on the method proposed by Nakamura, Matsubara & Suto (1998) in which the method to measure the anisotropy of the two-point correlation function induced by the geometric distortion and the time evolution of clustering independent of the shape and amplitude of the underlying fluctuation spectrum. We calculate the variances of the estimators to examine how accurately we can measure their quantities from the real galaxy redshift surveys. We find that from the SDSS galaxy sample, the time evolution of bias is not constrained while the SCDM model ($\Omega_m = 1$, $\Omega_\Lambda$=0) can be ruled out for the fiducial cosmological model ($\Omega_m = 0.3$, $\Omega_\Lambda$=0.7).

## 1.1 Introduction

It is well known that the anisotropy of the galaxy clustering caused by the peculiar velocity can be used to measure the redshift-distortion parameter $\beta(z) = b(z)^{-1} d\ln D_1(z)/d\ln a(z)$, where $D_1(z)$ is the liner growth rate, $b(z)$ is the linear and time-dependent bias factor, and $a(z)$ is the scale factor. Recently, Pecock et al.(2001) measured $\beta$ with the quadrupole to monopole ratio of the two-point correlation function from the galaxy redshift catalog of the 2dF Galaxy Redshift Survey (2dFGRS). They reported the resulting redshift-distortion parameter $\beta = 0.43 \pm 0.07$. The accuracy of the measure of this parameter will be higher when the 2dFGRS and the Sloan Digital Sky Survey (SDSS) are completed.

In addition to the peculiar velocity, for the nonzero depth of the survey the geometric distortion distort the clustering pattern. The geometric distortion in such a depth of these galaxy redshift surveys is weak. However de Laix & Starkman (1998) suggested the possibility of measuring the geometric distortion and constraining the cosmological parameters without the external constraints assuming the surveys of approximate depth of the SDSS. In their analysis, the quadrupole to monopole ratio of the power spectrum has been adopted. Here we adopt the method proposed by Nakamura, Matsubara & Suto (1998). Their method does not need any assumption in the shape of the underlying fluctuation spectrum although the direct fit using the two-point correlation function or power spectrum does need. Therefore, by this method we can get the clean signal from the redshift-space distortions. However, how well this method work in real galaxy redshift surveys is not examined. In this paper, We test this method by variance calculations of the estimators.

## 1.2 Two-point Correlation Function in Redshift Space

For the samples from the nearby galaxy redshift surveys, radial distances of galaxies are usually determined with the Hubble's law, $s = z/H_0$. Then in this paper we define the redshift-space radial coordinate to be the coordinate determined by $s = z/H_0$ from the observed redshift $z$. Then in redshift space, the separation $R$ of the two points located at redshift $z$, and separated by $\Delta z(\ll z)$ and angle $\Delta\theta(\ll 1)$ are written as

$$R = [(\Delta z/H_0)^2 + \{(z/H_0)\Delta\theta\}^2]^{1/2}, \tag{1.1}$$

and the cosine of the angle between the difference vector of the two points and the line of sight direction is

$$\mu = (\Delta z/H_0)/R. \tag{1.2}$$

In redshift space, the two-point correlation function can be expanded up to the linear order in $z$ in terms of $R$ and $\mu$ as (Nakamura, Matsubara & Suto 1998),

$$\xi^s(R,\mu;z) \simeq \sum_{l=0}^{3} \zeta_{2l}(R;z)P_{2l}(\mu), \tag{1.3}$$

where $P_{2l}(\mu)$ is the Legendre polynomials and

$$\zeta_{2l}(R;z) \simeq \xi_{2l}(R) + z\eta_{2l}[R;\beta_0,q_0,\phi_0,\psi_0,\xi(R)]. \tag{1.4}$$

The first term of the right-hand side of equation (1.4) is the two-point correlation function at present and the second term describes the effect of the geometric distortion and the time evolution. $\xi_{2l}(R)$ and $\eta_{2l}(R)$ in equation (1.4) are expressed as

$$\xi_0(R) = \left[1+\frac{2}{3}\beta_0+\frac{1}{5}\beta_0^2\right]\xi(R) \tag{1.5}$$

$$\xi_2(R) = \left[\frac{4}{3}\beta_0+\frac{4}{7}\beta_0^2\right]\xi_d(R) \tag{1.6}$$

$$\xi_4(R) = \frac{8}{35}\beta_0^2\left[\xi_d(R)+\frac{7}{2}\xi_q(R)\right] \tag{1.7}$$

$$\begin{aligned}\eta_0(R) = & -\left[\left(1+\frac{1}{3}\beta_0\right)\phi_0+\left(\frac{1}{3}\beta_0+\frac{1}{5}\beta_0^2\right)\psi_0\right]\xi(R)\\ & -\frac{2}{3}\left[1+\frac{4}{5}\beta_0+\frac{9}{35}\beta_0^2\right]\frac{d\xi(R)}{d\ln R}\\ & \times(1+q_0)\end{aligned} \tag{1.8}$$

$$\begin{aligned}\eta_2(R) = & -\left[\frac{2}{3}\beta_0\phi_0+\left(\frac{2}{3}\beta_0+\frac{4}{7}\beta_0^2\right)\psi_0\right]\xi_d(R)\\ & +\left[\left(\frac{20}{7}\beta_0+\frac{4}{3}\beta_0^2\right)\xi_d(R)-\frac{1}{3}\left(1+\frac{26}{7}\beta_0+\frac{11}{7}\beta_0^2\right)\frac{d\xi(R)}{d\ln R}\right]\\ & \times(1+q_0)\end{aligned} \tag{1.9}$$

$$
\begin{aligned}
\eta_4(R) &= -\phi_0\xi_4(R) + \left[\left(\frac{8}{7}\beta_0 + \frac{24}{11}\beta_0^2\right)\xi_d(R)\right. \\
&\quad \left. + \frac{32}{11}\beta_0^2\xi_q(R) - \frac{8}{35}\left(\beta_0 + \frac{13}{11}\beta_0^2\right)\frac{d\xi(R)}{d\ln R}\right](1+q_0) \qquad (1.10) \\
\eta_6(R) &= \frac{4}{33}\beta_0^2\left[4\xi_d(R) + 9\xi_q(R) - \frac{2}{7}\frac{d\xi(R)}{d\ln R}\right](1+q_0) \qquad (1.11)
\end{aligned}
$$

where $\xi(R)$ is the real-space two-point correlation function of galaxies at present, $q_0$ is the deceleration parameter at present, $\beta_0$ is the redshift-distortion parameter at present, and $\phi_0$ and $\psi_0$ are defined as

$$
\begin{aligned}
\phi_0 &= 2f_0 - 2b_1, \qquad (1.12) \\
\psi_0 &= \frac{3\Omega_0}{f_0} - 2(1-q_0), \qquad (1.13)
\end{aligned}
$$

where $f_0 = f(z=0)$, $b_1 = d\ln b(z)/dz|_{z=0}$, $\xi_d(R)$ and $\xi_q(R)$ are defined in terms of $\xi(R)$ as

$$
\begin{aligned}
\xi_d(R) &= \xi(R) - \bar{\xi}(R), \qquad (1.14) \\
\xi_q(R) &= \bar{\xi}(R) - \bar{\bar{\xi}}(R), \qquad (1.15)
\end{aligned}
$$

where

$$\bar{\xi}(R) = \frac{3}{R^3}\int_0^R dRR^2\xi(R), \qquad (1.16)$$

$$\bar{\bar{\xi}}(R) = \frac{5}{R^5}\int_0^R dRR^4\xi(R). \qquad (1.17)$$

Performing the following integration

$$
\begin{aligned}
I_0(R) &= \left[\frac{3}{R^3}\int_0^R dR'R'^2 - \frac{5}{R^5}\int_0^R dR'R'^4\right]\eta_0(R'), \qquad (1.18) \\
I_2(R) &= \frac{3}{R^5}\int_0^R dR'R'^4\eta_2(R'), \qquad (1.19) \\
I_4(R) &= \frac{35}{4}R^2\int_R^\infty dR'R'^{-3}\eta_4(R'), \qquad (1.20)
\end{aligned}
$$

we find

$$Ax = d, \qquad (1.21)$$

where ${}^t x = [1+q_0, \phi_0, \psi_0]$, ${}^t d = [I_0(R), I_2(R), I_4(R)]$, and $A$ is the $3\times 3$ matrix whose elements are

$$
\begin{aligned}
A_{11} &= \frac{2}{3}\left[1 + \frac{4}{5}\beta_0 + \frac{9}{35}\beta_0^2\right]\left[2\xi_d(R) - 5\xi_q(R)\right], \qquad (1.22) \\
A_{12} &= -\left[1 + \frac{1}{3}\beta_0\right]\xi_q(R), \qquad (1.23) \\
A_{13} &= -\left[\frac{1}{3}\beta_0 + \frac{1}{5}\beta_0^2\right]\xi_q(R), \qquad (1.24) \\
A_{21} &= -\left[1 + \frac{26}{7}\beta_0 + \frac{11}{7}\beta_0^2\right]\xi_d(R) - \left[1 + 8\beta_0 + \frac{25}{7}\beta_0^2\right]\xi_q(R), \qquad (1.25)
\end{aligned}
$$

$$
\begin{aligned}
A_{22} &= \beta_0 \xi_q(R), && (1.26)\\
A_{23} &= \left[\beta_0 + \frac{6}{7}\beta_0^2\right] \xi_q(R), && (1.27)\\
A_{31} &= 2\left[\beta_0 + \frac{13}{11}\beta_0^2\right] \xi_d(R) + \frac{40}{11}\beta_0^2 \xi_q(R), && (1.28)\\
A_{32} &= 0, && (1.29)\\
A_{33} &= -\beta_0^2 \xi_q(R). && (1.30)
\end{aligned}
$$

## 1.3 The Estimator

Here we consider the following estimator of the two-point correlation function instead of binning the data,

$$
\xi_z^{\mathrm{est}}(R,\mu) = \frac{DD_z^\alpha - DR_z^\alpha + RR_z^\alpha}{RR}, \tag{1.31}
$$

where

$$
\begin{aligned}
DD_z^\alpha &= \frac{1}{2}\sum_{ij} w_{ij} z_{ij}^\alpha n_i n_j \Theta_{ij}^{R,\mu}, && (1.32)\\
DR_z^\alpha &= \frac{1}{2}\sum_{ij} w_{ij} z_{ij}^\alpha n_i n_j^{\mathrm{r}} \Theta_{ij}^{R,\mu}, && (1.33)\\
RR_z^\alpha &= \frac{1}{2}\sum_{ij} w_{ij} z_{ij}^\alpha n_i^{\mathrm{r}} n_j^{\mathrm{r}} \Theta_{ij}^{R,\mu}, && (1.34)
\end{aligned}
$$

where $z_{ij} = (z_i + z_j)/2$, $\alpha = 0$ or $1$, $n_i$ is the galaxy number density and $w_{ij}$ is the pair weighting. We define the following quantity,

$$
\zeta_{2l}^\alpha(R) = \frac{4l+1}{2}\int_{-1}^{1} d\mu \xi_z^\alpha(R,\mu) P_{2l}(\mu). \tag{1.35}
$$

The ensemble average of equation (1.35) reduces to the following form

$$
\langle \zeta_l^\alpha(R)\rangle \simeq c_1^\alpha \xi_l(R) + c_2^\alpha \eta_l(R). \tag{1.36}
$$

$\xi_l(R)$ and $\eta_l(R)$ are estimated with $\zeta_l^0(R)$ and $\zeta_l^1(R)$. Then the inversion of equation (1.21) yields the measure of $q_0$, $\phi_0$ and $\psi_0$. Consequently we can estimate $\Omega_m$, $\Omega_\Lambda$, $b_0$ and $b_1$.

## 1.4 Results and Conclusions

We performed the variance calculations adopting th estimator (1.31). We assume the main galaxy sample in the SDSS and the LCDM model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) for the fiducial cosmological model. We find that the time evolution of bias (parameter $b_1$) is not constrained while the SCDM model ($\Omega_0 = 1$, $\Omega_\Lambda$=0) can be ruled out for the fiducial LCDM model (see Fig. 1.1). However more precise analyses using the mock sample from a N-body simulation are needed because our analyses presented in the present paper has been performed in the linear regime.

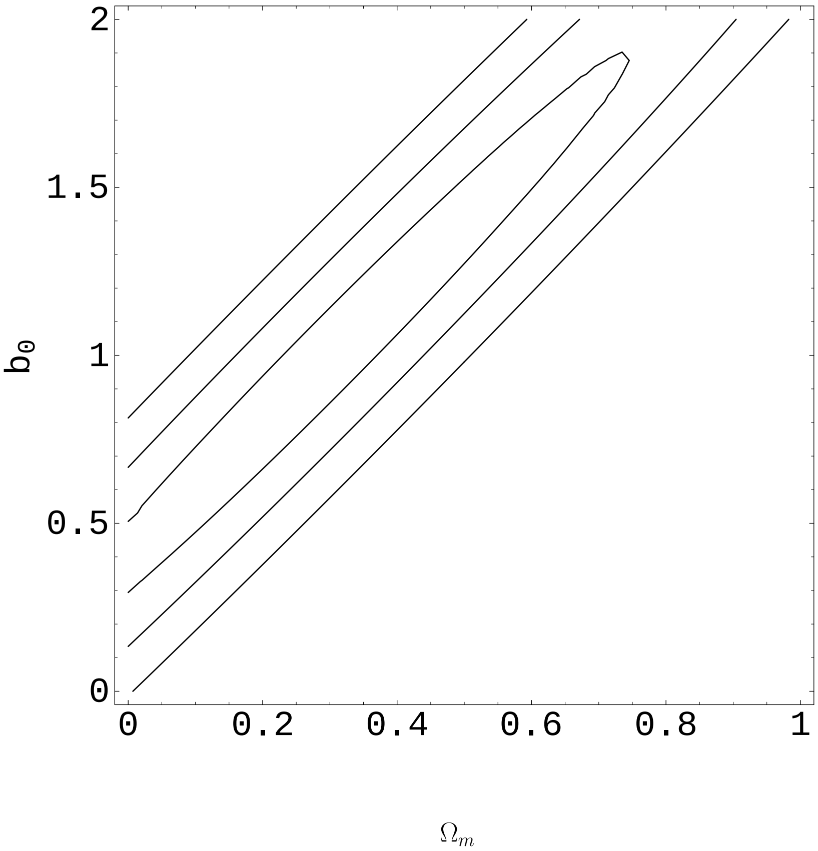


Fig. 1.1. The $1\sigma$, $2\sigma$ and $3\sigma$ confidence levels for $\Omega_m$ and $b_0$.

## References

de Laix, A. A., & Starkman, G. D. 1998, ApJ, 510, 427
Nakamura, T. T., Matsubara, T., & Suto, Y. 1998, ApJ, 494, 13
Peacock, J. A., et al. 2001, Nature, 410, 169